

Dt 12.12.19

Q What is a machine

Ans machine is a contrivance does some use full work by availing some energy from other source

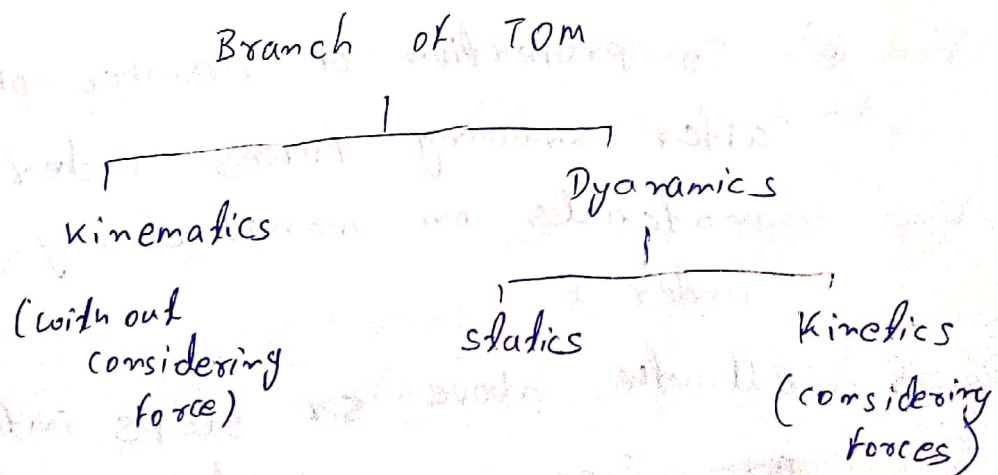
OR

A machine is a device and if we give some input to it, it give some output. It can not run by it self.

$$\text{Power}^{(Z)} = \frac{2\pi NIT}{60}$$

Dt 13.12.19

SIMPLE MECHANISM



scope of subject theory of machine.

A design of a machine is carried out in the following six steps:-

- ① To know the purpose for which the machine is required to be used.
- ② To study the motion required to make the machine to the purpose started on the one
- ③ To select the contrivance to produce the required motion under ~~to~~ 2
- ④ To calculate the forces (static and dynamic) acting in the members constituting the contrivance under ~~the~~ 3
- ⑤ To select suitable material for manufacture of the members under ~~the~~ 4
- ⑥ To proportion of members of the machine after knowing forces under 4 and materials on manufacturing and under 5

All the above six steps inter related. The steps 2, 3, and 4 include the theory of machine.

Include 5 and 6 include the design of machine of elements.

steps number 2 include kinematics.

Resistance body :-

The body which does not suffer appreciable change in physical form by the force acting on it. ~~They~~ ^{It} did not be ~~resist~~ ^{rigid} and as such it includes elastic body, such as spring, belts, as well as fluids in hydraulic press.

Link or Element

A link is a resistance body or assembly of resistance body which constituted part or parts of the machine connecting other parts which have motion relative to it.

OR

A link consist of number of parts, connected in such a way that they form one unit and have no motion relative to each other.

Example → piston, piston rod and cross-head.

→ crank pin, crank shaft and

→ cross-head, connecting rod, crank and frame

Classification of links

- ① Rigid links Ex connecting rod, crank
- ② Flexible links Ex spring, belt
- ③ fluid links Ex hydraulic press.

Kinematic pair

Two link or elements make a pair, which have relative motion between them.

Types of kinematic pair

① According to motion.

② ① sliding pair

③ ② Turning pair

④ ③ Rolling pair

⑤ ④ screw pair

⑥ ⑤ spherical pair

Assignment - I

Give the examples of the above five types of pair.

② According to contact surface

① Lower pair

② Higher pair

① Lower pair

When two elements have surface contact while in motion, the pair so formed is known as lower pair. In this case the relative motion is purely turning or sliding. Ex. shaft revolving in a bearing, straight line motion mechanism.

steering gear, universal joint

(ii) Higher pair

When two elements have point or line contact while in motion, the pair so formed is known as higher pair. In this case the motion is ^{both} sliding and turning. Ex belt, chain & sprocket.

(3) According to forces applied

(i) closed pair

(ii) Unclosed pair

(i) closed pair

When two elements of a pair are held together mechanically it forms a closed pair. All lower pairs are closed pairs.

(ii) unclosed pair

When two elements of a pair are not held together by mechanically it forms an unclosed pair.

Kinematic chain

When kinematic pairs are so coupled that the last link is joined to the first link to transmit a definite completely constant motion it forms a kinematic chain

$$L = 2P - 4$$

$$L = \frac{2}{3}(J + 2)$$

L = Link

P = Pairs

J = joints.

Types of motion

- ① completely constrained motion
- ② partially constrained motion
- ③ In completely constrained motion

① completely constrained motion

The motion takes place in a definite direction

② Partially constrained motion

when the constrained motion is not completed by it self but by some other means. Ex foot step bearing, rotor in a

vertical turbine

③ Incompletely constrained motion.

when the motion take place in more than one direction

Mechanism

when number of pair are so connected that one of it's link and other move relative to each other. It is called a mechanism.

There are as many mechanism as
number of links
there are n link

Types of mechanism

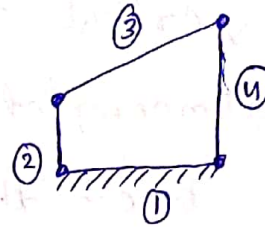
① simple Ex. 4 bar chain

② compound Ex. more than 4 bar

③ ~~Simple~~

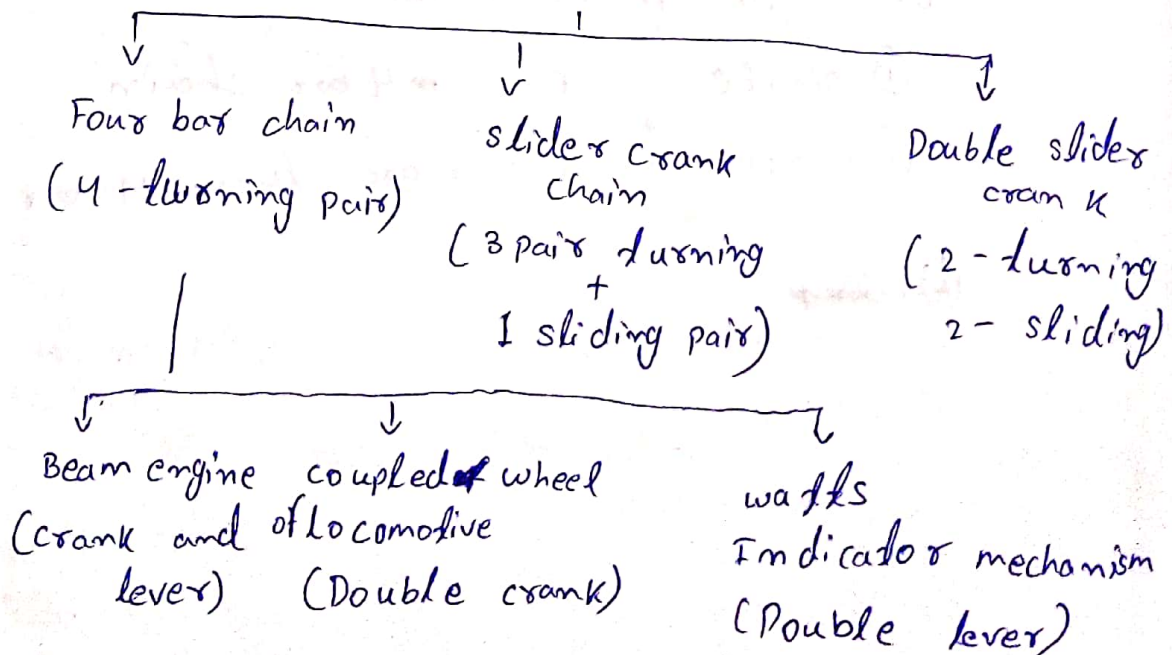
Inversion

If in a kinematic chain one link is fixed and other move relative to each other that is called a mechanism.

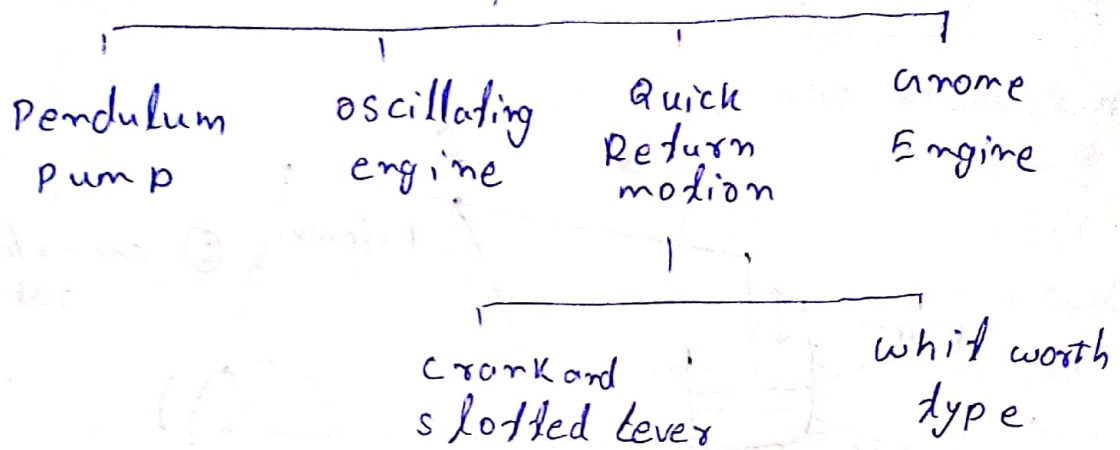


Let us take this mechanism is the original mechanism and if another link is fixed of the kinematic chain and other move relative to each other, that is called another mechanism and this mechanism is called the inversion of the original mechanism.

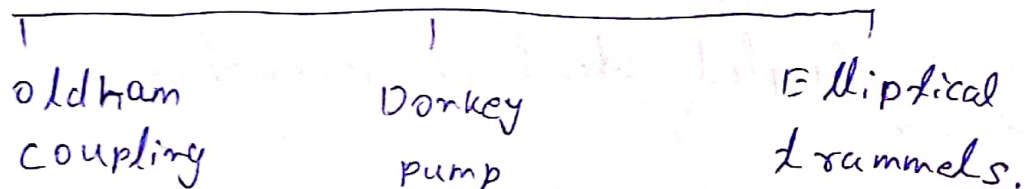
Types of ~~mechanism~~ kinematic chain



slider crank chain

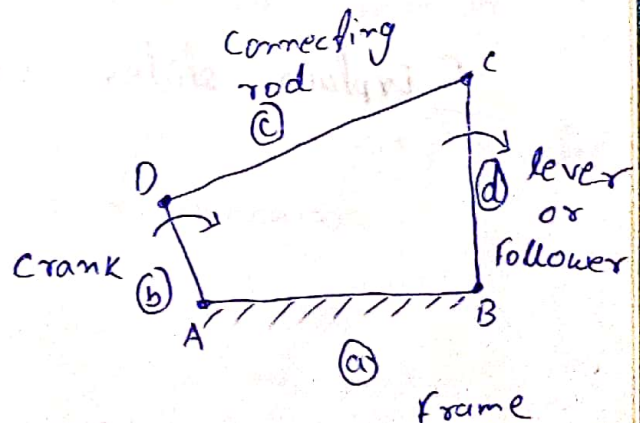


Double slider crank



Four bar chain

Q write short notes
on four bar chain
mechanism or
Quadrilateral cycle chain

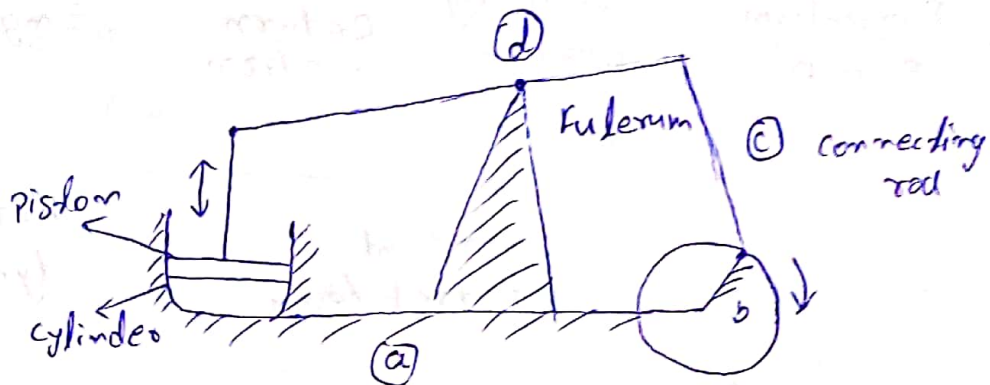


Grashof's rule

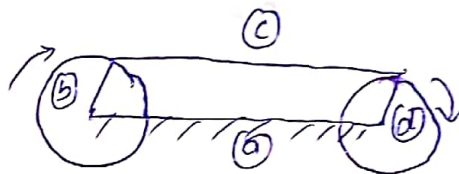
The sum of the length of the small and largest link should not be ~~greater~~ ^{greater} than the sum of the length of the other two links.

$$AD + CD \not> AB + BC$$

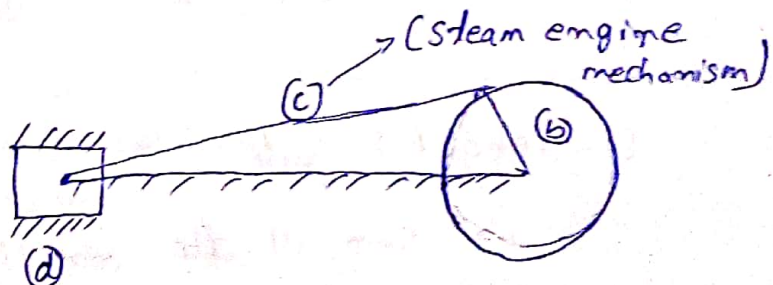
Beam Engine



Coupled wheel of locomotive



Q Explain slider crank chain mechanism



Difference between machine and mechanism

machine

→ It is an assembly of number of links having relative motion and capable of transforming available energy into certain useful work

→ It is ~~constituted~~ ^{constituted} ~~consist~~ of mechanism meant for transmitting energy to do useful work

→ It is a practical development of any mechanism

mechanism

→ It is also an assembly of number of links and motion through one of them producing definite motion to other but not transmitting any force.

→ A mechanism is not ~~necessarily~~ ^{necessarily} a machine. It is a skeleton of machine which is not meant to transmit energy but to provide definite motion.

→ The working model of machine

DL-17.12.19.

Cam and follower

A cam is defined as a rotating or reciprocating element of a mechanism which imparts rotating, reciprocating or oscillating motion to another element known as follower.

In most of cases cam connected with a frame forming a turning and follower is connected with a frame to form a sliding pair.

The cam and follower form a three link mechanism of higher pair type.

link - 1 Drives, cam has a curved or straight contact surface

link - 2 Driven, follower and gets motion with contact with cam

link - 3 frame which is used to support the cam and guide the follower.

Types of cam

- ① Radial or Disc type
- ② cylindrical

- ① Radial or Disc type.
- ② Reciprocating
- ③ Tangent
- ④ Circular

Types of follower

- | | |
|-------------------|--|
| ① Knife edged | } According to surface contact
} offset
} coinciding |
| ② Roller type | |
| ③ Flat faced | |
| ④ Spherical faced | |
| ⑤ Reciprocating | } According to motion |
| ⑥ Oscillating | |
| ⑦ Radial | } According to path of motion of follower. |
| ⑧ offset | |

Assignment - 2

- ① Draw the fig of all the types of cam and followers
- ② Explain 4 bar chain mechanism
- ③ Number of degree of freedom for plain mechanism

It is define as the number of input parameters (pair variable) which must be independently control in order to bring the mechanism into a useful engineering purpose

$$n = 3(l-1) - 2j - h$$

where

n = number of degree of freedom

l = number of links

j = number of joint

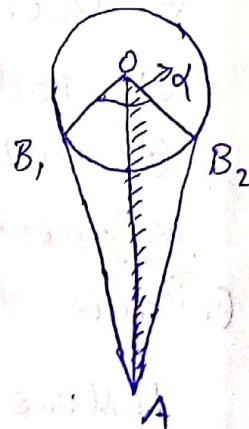
h = number of higher pair

Dt 18.12.19

Inversion of single slider crank chain

Crank and slotted lever mechanism

This type of mechanism is used in shaper machine



$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}}$$

$$= \frac{360 - \alpha}{\alpha}$$

Q A crank and slotted lever mechanism has a center distance of 300 mm between the center of oscillation of the slotted lever and the centre of rotation of crank.

The radius of the crank is 120 mm.

Find the ratio of the time of cutting to the time of return stroke

Given

$$AO = 300 \text{ mm}$$

$$OB_1 = 120 \text{ mm}$$

$$\sin\left(90 - \frac{\alpha}{2}\right) = \frac{OB_1}{OA}$$

$$\sin\left(90 - \frac{\alpha}{2}\right) = \frac{120}{300}$$

$$\sin\left(90 - \frac{\alpha}{2}\right) = 0.4$$

$$\cos \frac{\alpha}{2} = 0.4$$

$$\frac{\alpha}{2} = \cos^{-1}(0.4)$$

$$\frac{\alpha}{2} = 66.4218$$

$$\alpha = 66.4218 \times 2$$

$$= 132.8436$$

$$\text{The ratio} = \frac{\text{Time of cutting stroke}}{\text{Time of return stroke}}$$

$$= \frac{360 - \alpha}{\alpha}$$

$$= \frac{227.1564}{132.8436}$$

$$= 1.71$$

Chapter 2

Friction

18.12.19

It is an opposing force which opposes the motion of the body in which a body moves or tends to move.

Different terms Assignment - 3

- ① Friction (f)
- ② Normal reaction (R_N)
- ③ ~~co~~ co-efficient of friction ($\frac{f_L}{R_N}$)
- ④ Limiting friction (f_L)
- ⑤ Angle of friction (ϕ)
- ⑥ Angle of repose (α)



Dt 19.12.19

Friction between nut and ~~screw~~ screw

Ex Screw jack

$$\text{Mean Diameter } D_m = \frac{D_o + D_i}{2}$$

$$D_m = D_o - \frac{p}{2}$$

$$D_i = D_o - p$$

D_o = outer diameter

D_i = Internal diameter

Terms related to screw thread

① helix angle (θ)

② Pitch

The axial distance between two thread

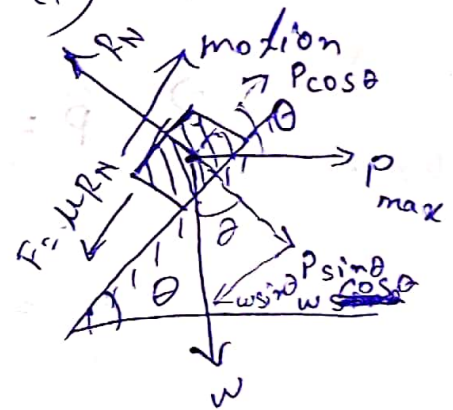
③ Lead

The distance moved by the screw in one revolution.

single start (Pitch = Lead)

Double start (2 pitch = Lead)

multiple start (n Pitch = Lead)



$$\mu R_N = F, R_N$$

$$P \cos \theta, P \sin \theta$$

$$W \cos \theta, W \sin \theta$$

$$\sum H = 0$$

$$R = \sqrt{(\sum H)^2 + (\sum V)^2}$$

$$\Rightarrow \mu R_N + W \sin \theta = P \cos \theta \quad \text{--- (1)}$$

$$\sum V = 0$$

$$\Rightarrow R_N = P \sin \theta + W \cos \theta \quad \text{--- (2)}$$

Putting the value of R_N in eqⁿ (1)

$$\mu (P \sin \theta + W \cos \theta) + W \sin \theta = P \cos \theta$$

$$\frac{\sin \phi}{\cos \phi} (P \sin \theta + W \cos \theta) + W \sin \theta = P \cos \theta$$

$$\sin \phi (P \sin \theta + W \cos \theta) + W \sin \theta \times \cos \phi = P \cos \theta \cos \phi$$

$$\Rightarrow P \sin \theta \cdot \sin \phi + W \cos \theta \cdot \sin \phi + W \sin \theta \cdot \cos \phi = P \cos \theta \cdot \cos \phi$$

$$\Rightarrow P (\sin \theta \cdot \sin \phi - \cos \theta \cdot \cos \phi) + W (\cos \theta \cdot \sin \phi + \sin \theta \cdot \cos \phi) = 0$$

$$\Rightarrow -P \cos (\theta + \phi) + W \sin (\theta + \phi) = 0$$

$$\Rightarrow W \sin (\theta + \phi) = P \cos (\theta + \phi)$$

$$\Rightarrow P = \frac{W \sin (\theta + \phi)}{\cos (\theta + \phi)}$$

$$= W \tan (\theta + \phi)$$

force required to raise the load

$$P = W \tan (\theta + \phi)$$

where

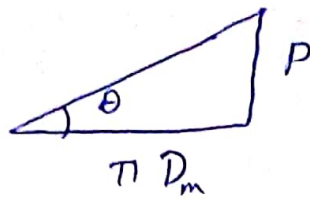
W = load to be lifted

θ = helix angle

$\phi = \tan^{-1}(\mu)$

effort required to lower the load

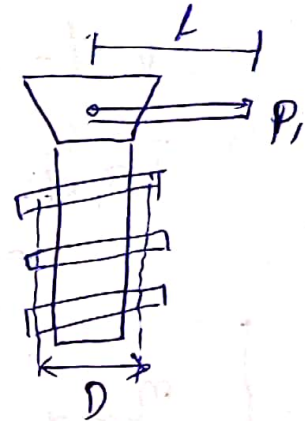
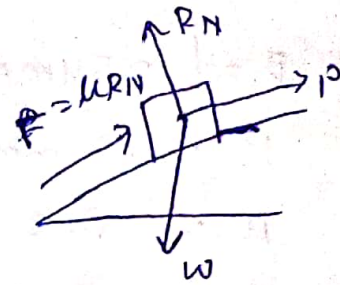
$$P = W \tan (\theta - \phi)$$



$$\tan \theta = \frac{P}{\pi D_m}$$

$$P_1 \times L = P \times \frac{d}{2}$$

$$P_1 = \frac{P \times d}{2L}$$



The mean diameter of a square threaded screw jack is 50 mm. The pitch of the thread is 12 mm. The coefficient of friction is 0.2. What force must be applied at the end of a handle of length 0.8 m long which is $\perp$ to the longitudinal axis of the screw to raise a load of 21 kN and to lower it.

Efficiency of a screw jack

$$\eta = \frac{\tan \theta}{\tan (\theta + \phi)}$$

$$= \frac{\text{Ideal effort}}{\text{actual effort}}$$

Prove

$$\eta_{\max} = \frac{1 - \sin \phi}{1 + \sin \phi} \quad \frac{d\eta}{d\phi} = 0$$

DL - 20.12.19

$$D = d = 54 \text{ mm}$$

$$P = 12 \text{ mm}$$

$$\mu = 0.2$$

$$L = 0.8 \text{ m} = 800 \text{ mm}$$

$$P \times L = \frac{P \times D}{2}$$

$$\left[P \times L = \mu [\tan (\theta \pm \phi)] \times \frac{D}{2} \right]$$

$$\eta_{\max} = \frac{1 - \sin \phi}{1 + \sin \phi}$$

$$(\alpha = \theta)$$

$$\eta = \frac{\tan \alpha}{\tan (\alpha + \phi)}$$

$$= \frac{\frac{\sin \alpha}{\cos \alpha}}{\frac{\sin (\alpha + \phi)}{\cos (\alpha + \phi)}}$$

$$= \frac{2 \sin \alpha \cdot \cos (\alpha + \phi)}{2 \cos \alpha \cdot \sin (\alpha + \phi)}$$

$$= \frac{\sin (\alpha + \phi) + \sin (\alpha - \phi)}{\sin (\alpha + \phi) - \sin (\alpha - \phi)}$$

~~$$\frac{\sin (\alpha + \phi) + \sin (\alpha - \phi)}{\sin (\alpha + \phi) - \sin (\alpha - \phi)}$$~~

A load of 10 kN is to be lifted by means of a screw jack of 12 mm pitch and 50 mm outer diameter if a force of 100 N is applied at the end of a lever to raise the load. What should be the length of the lever is used coefficient of friction is 0.15. What is the mechanical advantage, state whether the screw is self locking.

Given

$$W = 10 \text{ kN} = 10 \times 10^3 \text{ N}$$

$$P = 12 \text{ mm}$$

$$D_o = 50 \text{ mm}$$

$$P_1 = 100 \text{ N}$$

$$\mu = 0.15$$

$$\begin{aligned} D_m &= D_o - \frac{P}{2} \\ &= 50 - \frac{12}{2} \\ &= 50 - 6 \\ &= 44 \text{ mm} \end{aligned}$$

$$\begin{aligned} P &= W \tan (\alpha + \phi) \\ &= W (\tan \alpha + \tan \phi) \end{aligned}$$

$$\tan \alpha = \frac{P}{\pi D_m}$$

$$= \frac{12}{\pi \times 44}$$

$$= 0.0868$$

$$P = 10^4 \times (0.0868 \div 0.15)$$

$$= \frac{2368}{100} \text{ mm}$$

$$P_1 \times l = P \times \frac{D_m}{2}$$

$$l = \frac{P \times D_m}{2 \times P_1}$$

$$= \frac{2368}{2 \times 100} \times 44$$

$$= \frac{520.96}{100} \text{ mm}$$

$$M.A = \frac{\text{Load Lifted}}{\text{Effort applied}} = \frac{W}{P_1}$$

$$= \frac{10^4}{100}$$

$$= 100 \text{ N/mm}$$

$\phi < \alpha$ = over hauling condition
 $\phi > \alpha$ = self locking

condition of self locking

$$\eta < 50\%$$

$$\eta = \frac{\tan \alpha}{\tan \alpha + \tan \phi}$$

$$= \frac{0.0868}{0.0868 + 0.15}$$

$$= 0.3665$$

$$= 0.3665 \times 100$$

$$= 36.65\%$$

Let w = load lifted

p = effort

x = distance moved by load

y = distance moved by effort

Input output

$P \times y$

$w \cdot x$

$$\text{Loss} = P_y - w_x$$

~~reversible~~

~~Self locking then so~~

self locking

reversible

50

Point of reversible

$w_x > P_y - w_x$ (reversible)

or
point of self locking

$w_x < P_y - w_x$ (irreversible)

$$w_x < P_y$$

$$\frac{w_x}{P_y} < \frac{1}{2}$$

$$\Rightarrow \eta < \frac{1}{2} \text{ or } \eta < 50\%$$

Example 10.6, 10.8

DL 21.12.19

$$D_m = 54 \text{ mm}$$

$$P = 12 \text{ mm}$$

$$\mu = 0.2$$

$$L = 0.8 \text{ m} = 8 \times 100 = 800 \text{ mm}$$

$$W = 21 \text{ kN} = 21 \times 10^3 \text{ N}$$

$$\theta = \tan^{-1} \left(\frac{P}{4 D_m} \right)$$

$$= \tan^{-1} \left(\frac{12}{\pi \times 54} \right)$$

$$= \tan^{-1} (0.0707)$$

$$= 4.044$$

$$\phi = \tan^{-1} (\mu)$$

$$= \tan^{-1} (0.2)$$

$$= (11.3099)$$

$$= 11.31$$

$$P_{\text{raised}} = W \tan (\theta + \phi)$$

$$= 21 \times 10^3 \times \tan (4.044 + 11.31)$$

$$= 5766.228 \text{ N}$$

$$= 5.766 \text{ kN}$$

$$P_{\text{lower}} = W \tan (\theta - \phi)$$

$$= 21 \times 10^3 \times \tan (\theta - \phi)$$

$$= 21 \times 10^3 \times \tan (\phi - \theta)$$

$$= 21 \times 10^3 \times \tan (11.31 - 4.044)$$

$$= 2677.49 \text{ N}$$

$$= 2.677 \text{ kN}$$

Force required to raise the load at the end of the lever.

$$P_1 \times l = P \times d/2$$

$$P_1 = \frac{P \times d}{2l}$$

$$= \frac{5766.228 \times 54}{2 \times 800}$$

$$= 194.61 \text{ N}$$

Force required to lower the load at the end of the lever

$$P_1 \times l = P \times d/2$$

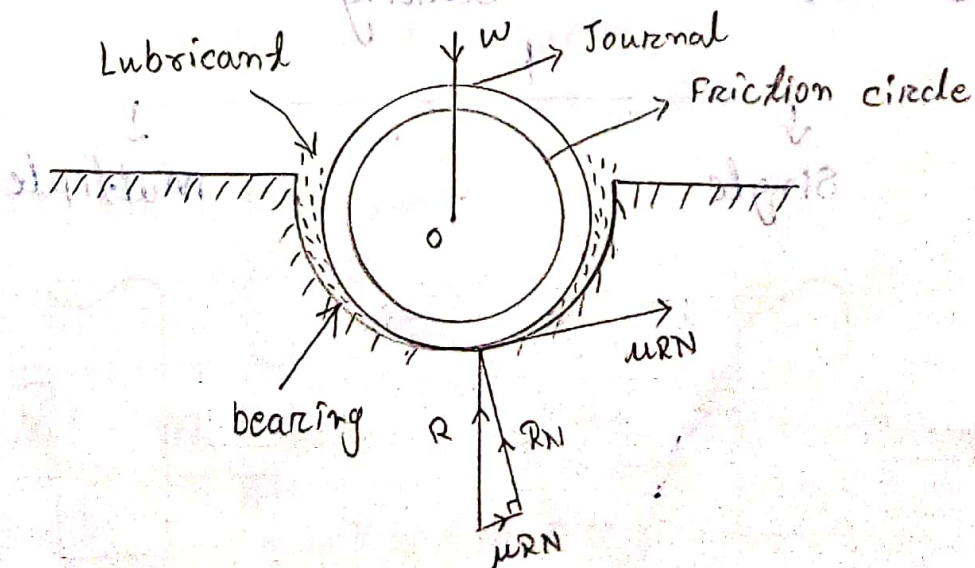
$$P_1 = \frac{P \times d}{2l}$$

$$= \frac{2677.49 \times 54}{2 \times 800}$$

$$= 90.36 \text{ N}$$

Friction on Bearing

Dl



The Fixed outer element of turning pair is called bearing.

The rotating element (position of the inner element) which fits in the outer bearing is called journal.

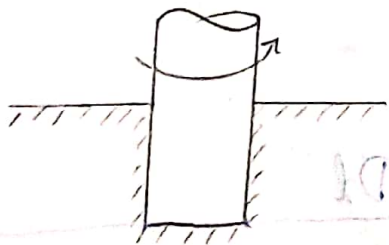
Types of bearing

- ① Roller bearing
- ② Needle Roller bearing
- ③ Ball bearing

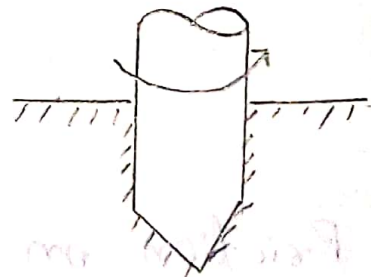
Classification On the basis of shape and size

- ① Pivot bearing

Flat
pivot bearing

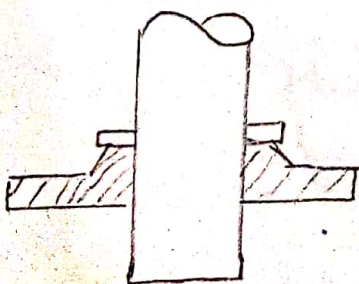


Conical
pivot bearing

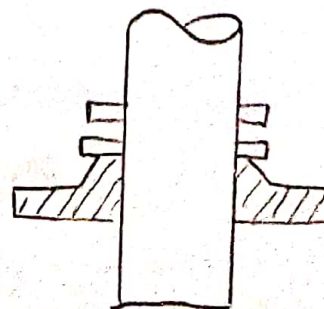


- ② Flat collar bearing

Single



Multiple

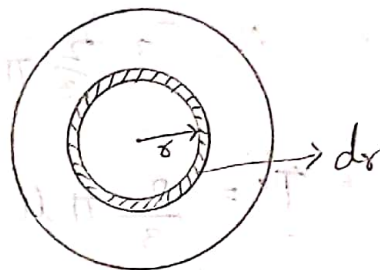
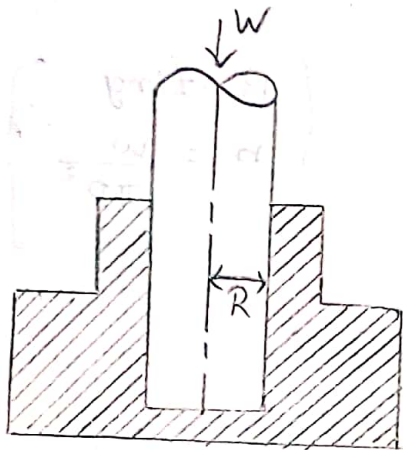


Torque transmitted in flat pivot for bearing

(a) Uniform pressure condition

(b) Uniform wear condition.

For Uniform Pressure



Let W = load transmitted over bearing surface

R = Radius of bearing surface

p = Intensity of pressure for unit area.

μ = Coefficient of friction.

Consider a small ring of radius ' r ' and thickness dr .

Area of ring = $(2\pi r)dr$

Load transmitted to the ring

$$\begin{aligned} \int W &= p \times \text{Area} \\ &= p \times 2\pi r \cdot dr \end{aligned}$$

Frictional resistance

$$F = \mu \cdot \int W$$

$$= \mu \cdot p \times 2\pi r \cdot dr$$

Total torque

$$T = \int_0^R 2\pi \mu p r^2 dr$$

$$T = 2\pi\mu p \int_0^R r^2 dr$$

$$= 2\pi\mu p \left[\frac{r^3}{3} \right]_0^R$$

$$= 2\pi\mu p \frac{R^3}{3}$$

$$= \frac{2}{3} \pi \mu p R^3$$

$$\left(\begin{array}{l} \text{Putting} \\ p = \frac{W}{\pi R^2} \end{array} \right)$$

$$T = \frac{2}{3} \pi \mu \frac{W}{\pi R^2} \cdot R^3$$

$$\boxed{T = \frac{2}{3} \mu W R}$$

For Uniform wear Condition

$$p \cdot r = C$$

$$p = \frac{C}{r}$$

We know

$$dW = p \times dA$$

$$= p \times 2\pi r \cdot dr$$

$$= \frac{C}{r} \times 2\pi r \cdot dr$$

$$= 2\pi C \cdot dr$$

$$W = \int_0^R dW$$

$$= \int_0^R 2\pi C \cdot dr$$

$$= 2\pi C \left[r \right]_0^R$$

$$W = 2\pi CR$$

$$C = \frac{W}{2\pi R}$$

Frictional Resistance

$$F = \mu \cdot \int W$$

$$= 2\pi \mu C \cdot d\delta$$

Frictional ~~Resistance~~ Torque

$$T_\delta = F \cdot \delta$$

$$= 2\pi \mu C \delta \cdot d\delta$$

Total Torque

$$T = \int_0^R T_\delta$$

$$= \int_0^R 2\pi \mu C \delta \cdot d\delta$$

$$= 2\pi \mu C \int_0^R \delta d\delta$$

$$= 2\pi \mu C \left[\frac{\delta^2}{2} \right]_0^R$$

$$= 2\pi \mu \frac{W}{2\pi R} \cdot \frac{R^2}{2}$$

$$\boxed{T = \frac{1}{2} \mu W R}$$

Assignment

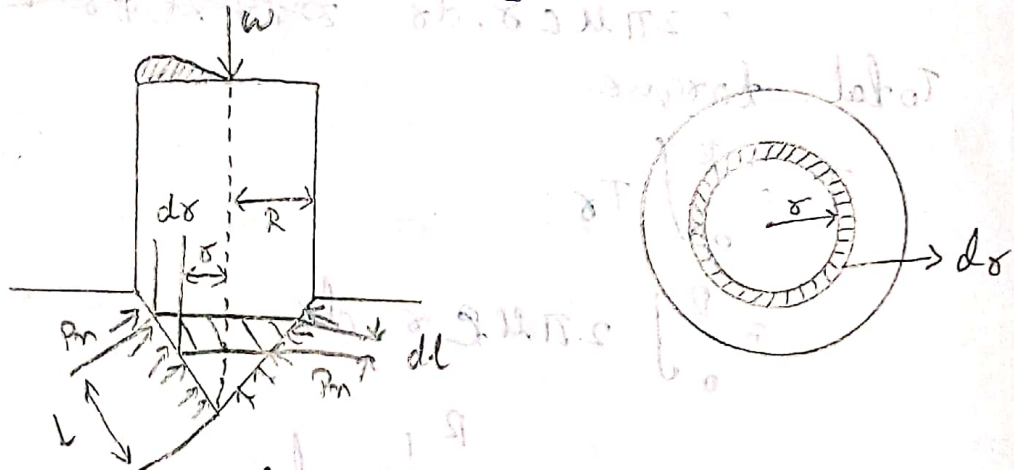
A vertical shaft 200 mm in diameter rotating at 130 RPM rest on a flat end foot step bearing shaft carries a

load of 28 kN. Assuming uniform pressure and coefficient of friction 0.04. Estimate power loss in friction.

Dt 13.01.2020

Torque Transmitted on Conical Pivot Bearing

(i) For Uniform Pressure



Let P_n = Intensity of pressure normal to the cone

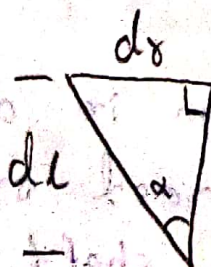
α = semi angle of cone

μ = coeff. of friction

R = Radius

d_o = thickness of ring

dl = small length



$$\sin \alpha = \frac{d_o}{dl}$$

$$dl = \frac{d_o}{\sin \alpha}$$

$$dl = dr \operatorname{cosec} \alpha$$

$$\begin{aligned} \text{Area of ring} &= 2\pi r \cdot dl \\ &= 2\pi r \cdot dr \operatorname{cosec} \alpha \end{aligned}$$

Normal load acting on the ring

$$\begin{aligned} \delta W_n &= \text{Normal Pressure} \times \text{area} \\ &= P_m \times 2\pi r \cdot dr \operatorname{cosec} \alpha \end{aligned}$$

Vertical load acting on the ring

$$\begin{aligned} \delta W &= \text{Vertical component of } \delta W_n \\ &= \delta W_n \times \sin \alpha \\ &= P_m \times 2\pi r dr \times \sin \alpha \times \frac{1}{\sin \alpha} \\ &= P_m 2\pi r dr \end{aligned}$$

Total vertical load

$$\begin{aligned} W &= \int_0^R \delta W \\ &= \int_0^R P_m \cdot 2\pi r \cdot dr \\ &= P_m 2\pi \int_0^R r dr \\ &= P_m 2\pi \left[\frac{r^2}{2} \right]_0^R \\ &= P_m 2\pi \frac{R^2}{2} \end{aligned}$$

$$W = P_m \pi R^2$$

$$P_m = \frac{W}{\pi R^2}$$

Frictional force on ring

$$F_r = \mu \sum W_m$$

$$= \mu P_m 2\pi r \cdot dr \cdot \operatorname{cosec} \alpha$$

Frictional Torque

$$T_r = F_r \cdot r$$

$$= \mu P_m 2\pi r^2 dr \cdot \operatorname{cosec} \alpha$$

Total Torque

$$T = \int_0^R T_r$$

$$= \mu P_m 2\pi \left[\frac{r^3}{3} \right]_0^R \operatorname{cosec} \alpha$$

$$= \mu P_m 2\pi \frac{R^3}{3} \operatorname{cosec} \alpha$$

$$= \mu \frac{W}{\pi R^2} 2\pi \frac{R^3}{3} \operatorname{cosec} \alpha$$

$$= \mu W \cdot \frac{2}{3} R \operatorname{cosec} \alpha$$

$$\boxed{T = \frac{2}{3} \mu W R \cdot \operatorname{cosec} \alpha}$$

(ii) Uniform wear condition

$$P_r \cdot r = C$$

$$P_r = \frac{C}{r}$$

$$\begin{aligned}\int \omega &= P_r \cdot 2\pi r \cdot dr \\ &= \frac{C}{r} \cdot 2\pi r \cdot dr \\ &= 2\pi C \cdot dr\end{aligned}$$

$$\omega = \int_0^R \int \omega$$

$$= \int_0^R 2\pi C \cdot dr$$

$$= 2\pi C [r]_0^R$$

$$= 2\pi C R$$

$$C = \frac{\omega}{2\pi R}$$

$$T_r = 2\pi \mu P_r \cdot r^2 \cdot \operatorname{cosec} \alpha \cdot dr$$

$$T = \int_0^R T_r$$

$$= \int_0^R 2\pi \mu P_r \cdot r^2 \cdot \operatorname{cosec} \alpha \cdot dr$$

$$= \cancel{2\pi \mu P_r} \cdot \cancel{\operatorname{cosec} \alpha} \cdot \cancel{r^2}$$

$$= \int_0^R 2\pi \mu \frac{C}{r} \cdot r^2 \cdot \operatorname{cosec} \alpha \cdot dr$$

$$= 2\pi \mu C \operatorname{cosec} \alpha \left[\frac{r^2}{2} \right]_0^R$$

$$= \cancel{2} \pi \mu C \operatorname{cosec} \alpha \left(\frac{R^2}{\cancel{2}} \right)$$

$$= \pi R^2 \mu C \operatorname{cosec} \alpha \cdot \frac{1}{2}$$

$$= \cancel{\pi} R^2 \mu \frac{\omega}{\cancel{2\pi R}} \operatorname{cosec} \alpha$$

$$T = \frac{1}{2} \mu W R \operatorname{cosec} \alpha$$

Formula derived

$$T = \frac{2}{3} \mu W R \operatorname{cosec} \alpha \text{ (uniform Pressure)}$$

$$T = \frac{1}{2} \mu W R \operatorname{cosec} \alpha \text{ (uniform wear)}$$

Q A conical Pivot bearing support a vertical shaft of 200 mm diameter it is subjected to a load of 35 kN. The angle of cone 120° . And the $\mu = 0.025$. Find the power lost in friction when the speed is 150 rpm

Given

$$D = 200 \text{ mm} \quad R = 100 \text{ mm} = 0.1 \text{ m}$$

$$W = 35 \text{ kN} = 35 \times 10^3 \text{ N}$$

$$2\alpha = 120^\circ \Rightarrow \alpha = \frac{120^\circ}{2} = 60^\circ$$

$$\mu = 0.025$$

$$N = 150$$

(i) Uniform Pressure

$$T = \frac{2}{3} \mu W R \operatorname{cosec} \alpha$$

$$= \frac{2}{3} \times 0.025 \times 35 \times 10^3 \times 0.1 \times \operatorname{cosec} 60^\circ$$

$$= \frac{2}{3} \times 0.025 \times 35 \times 10^3 \times 0.1 \times \frac{1}{\sin 60^\circ}$$

$$= 67.357 \text{ Nm}$$

$$P = T \cdot \omega$$

$$= 67.357 \times 35 \times 10^3$$

$$= 2357.495 \text{ J}$$

$$= 2357.495 \text{ Kg}$$

Assignment

Derive the formula for torque transmitted for flat pivot bearing under both condition

$$R \int_0^R = \frac{\sigma_1}{\sigma_2}$$

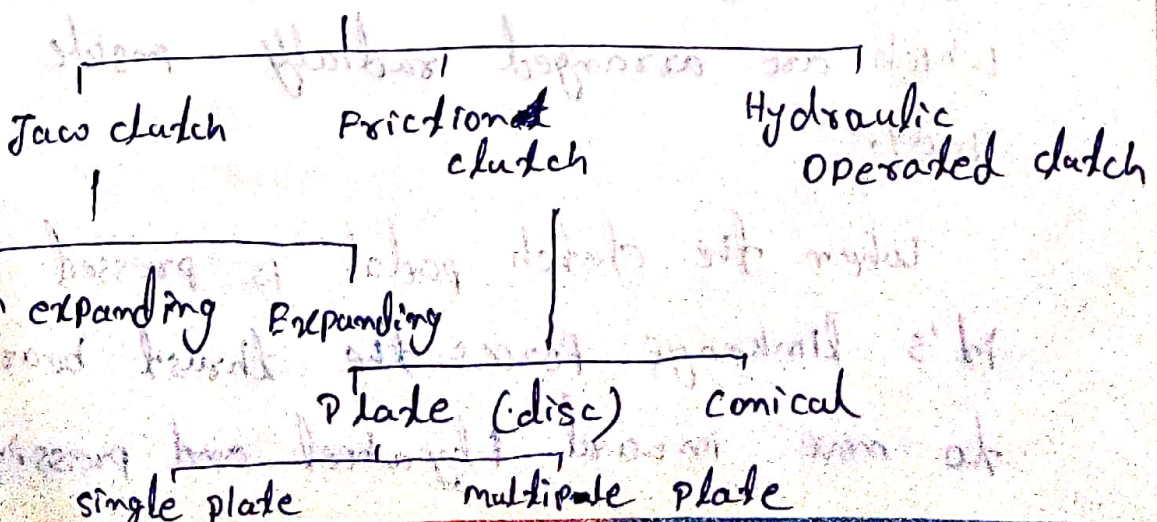
$$T = \frac{2}{3} \mu \omega \left(\frac{\sigma_1^3 - \sigma_2^3}{\sigma_1^2 - \sigma_2^2} \right) \quad \text{Uniform pressure}$$

$$T = \frac{1}{2} \mu \omega (\sigma_1 + \sigma_2) \quad \text{Uniform wear}$$

Dt 20.01.2020

Clutch

Clutch



Clutch is a device to connect driving and driven shaft where driven shaft can be disconnect almost instantaneously from the driving shaft is desired by the operator or driver

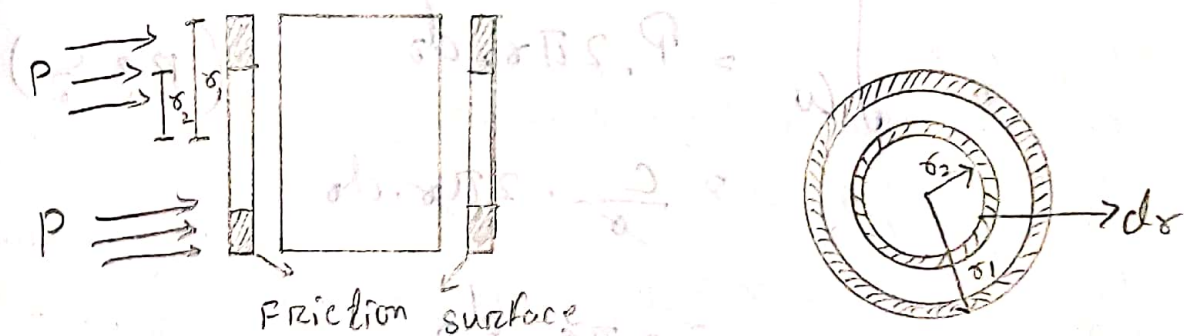
Working Principle of single clutch

It consist of a clutch plate whose both sides are face with friction material. It is mounted on the hub which is free to move ~~axially~~ axially along the splines of the driving shaft. The pressure plate is mounted inside the clutch body which is bolted to fly wheel. Both the pressure plate and fly wheel rotate with the engine crank shaft or driving shaft. The pressure plate pushes the clutch plate towards the fly wheel by means of a set of springs which are arranged radially inside the body.

When the clutch pedal is pressed down it's linkage forces the thrust bearing to move inward fly wheel and pressing the

longer ends of the lever inwards. On the other hand when the foot is taken up from the clutch pedal the thrust bearing moves back by the lever. These allow the springs to expand and thus pressure plate push the clutch plate back towards the fly wheel. The axial pressure exerted by the spring provide a frictional force and a torque is produced.

Torque transmitted by single clutch plate



Uniform Pressure Condition

We know $T_r = 2\pi \mu p r^2 dr$

Total torque

$$\begin{aligned}
 T &= \int_{r_2}^{r_1} T_r \\
 &= \int_{r_2}^{r_1} 2\pi \mu p r^2 dr \\
 &= 2\pi \mu p \left(\frac{r^3}{3} \right)_{r_2}^{r_1}
 \end{aligned}$$

$$T_0 = 2\pi\mu P \left[\frac{r^3}{3} \right]_{r_2}^{r_1}$$

$$= 2\pi\mu P \left(\frac{r_1^3 - r_2^3}{3} \right)$$

$$= \frac{2}{3} \pi \mu \omega \frac{(r_1^3 - r_2^3)}{r_1^2 - r_2^2} \left(P = \frac{\omega}{2\pi(r_1^2 - r_2^2)} \right)$$

$$= \frac{2}{3} \mu \omega \left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right)$$

Uniform wear condition

$$\int \omega = P \cdot 2\pi r \cdot dr \quad \left(P = \frac{C}{r} \right)$$

$$= \frac{C}{r} \cdot 2\pi r \cdot dr$$

$$= 2\pi C \, dr$$

Total work done

$$W = \int_{r_2}^{r_1} 2\pi C \, dr$$

$$= 2\pi C \left[dr \right]_{r_2}^{r_1}$$

$$= 2\pi C (r_1 - r_2)$$

$$C = \frac{W}{2\pi (r_1 - r_2)}$$

we know $T_r = 2\pi\mu C \cdot r \, dr$

Total torque

$$T = \int_{r_2}^{r_1} 2\pi \mu C r dr$$

$$= 2\pi \mu C \left[\frac{r^2}{2} \right]_{r_2}^{r_1}$$

$$= 2\pi \mu C \left(\frac{r_1^2 - r_2^2}{2} \right)$$

$$= \frac{2\pi \mu \omega}{2\pi(r_1 - r_2)} \times \left(\frac{r_1^2 - r_2^2}{2} \right)$$

$$= \frac{1}{2} \mu \omega \left(\frac{r_1^2 - r_2^2}{r_1 - r_2} \right)$$

$$= \frac{1}{2} \mu \omega \frac{(r_1 + r_2)(r_1 - r_2)}{(r_1 - r_2)}$$

$$= \frac{1}{2} \mu \omega (r_1 + r_2)$$

Imp

In case of problem on single plate clutch torque transmitted

$$T = \frac{n \cdot \mu \omega (r_1 + r_2)}{2} \quad \text{or} \quad (\text{uniform wear})$$

$$T = \frac{2n}{3} \mu \omega \frac{(r_1^3 - r_2^3)}{(r_1^2 - r_2^2)} \quad (\text{uniform pressure})$$

If both sides of the plate is effective
and $n = 2$

For multiple disc plate clutch

$$n = n_1 + n_2 - 1$$

where n_1 = number of disc plate on driving shaft

n_2 = number of disc plate into driven shaft

Assignment

Q₁ A single plate disc clutch with both sides effective has outer and inner diameters are 300 mm and 200 mm respectively. The maximum intensity of pressure at any contact surface is not to exceed 0.1 N/mm² if $\mu = 0.3$ determine the power transmitted by the clutch at a speed of 2500 RPM

Q₂ A multiple disc plate clutch has 3 disc on driving shaft and 2 on the driving shaft the out side diameters of the concave surface is 250 mm, and inside diameter is 120 mm, assuming uniform wear and $\mu = 0.3$. Find the maximum intensity of pressure between the disc for transmitting 5 kW at 1575 RPM

Frictional break

A break is a appliance by mean of which artificial frictional resistance is applied to a moving body to to it's motion or stop it's motion. The break absorb either kinetic energy of the moving member or potential energy given up by objects. The energy absorb by the breaks is dissipated in the form of heat. The capacity of the break is depend upon the following factors.

- ① Unit pressure between breaking surface
- ② high coefficient of friction between breaking surface
- ③ desi velocity of the break drum
- ④ Projected area of the friction surface.

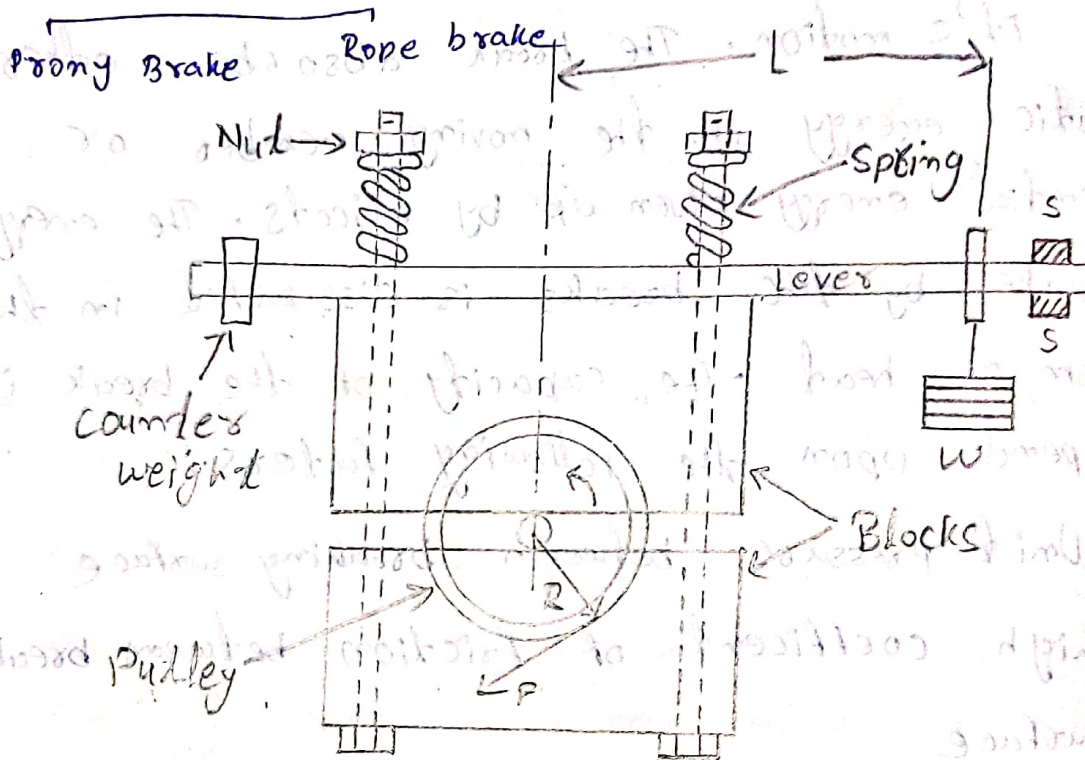
Material for breaking surface

- ① high coefficient of friction
- ② high wear rate
- ③ high heat dissipation capacity
- ④ Adequate mechanical strength
- ⑤ Should not be effected by oil, acid and moisture.

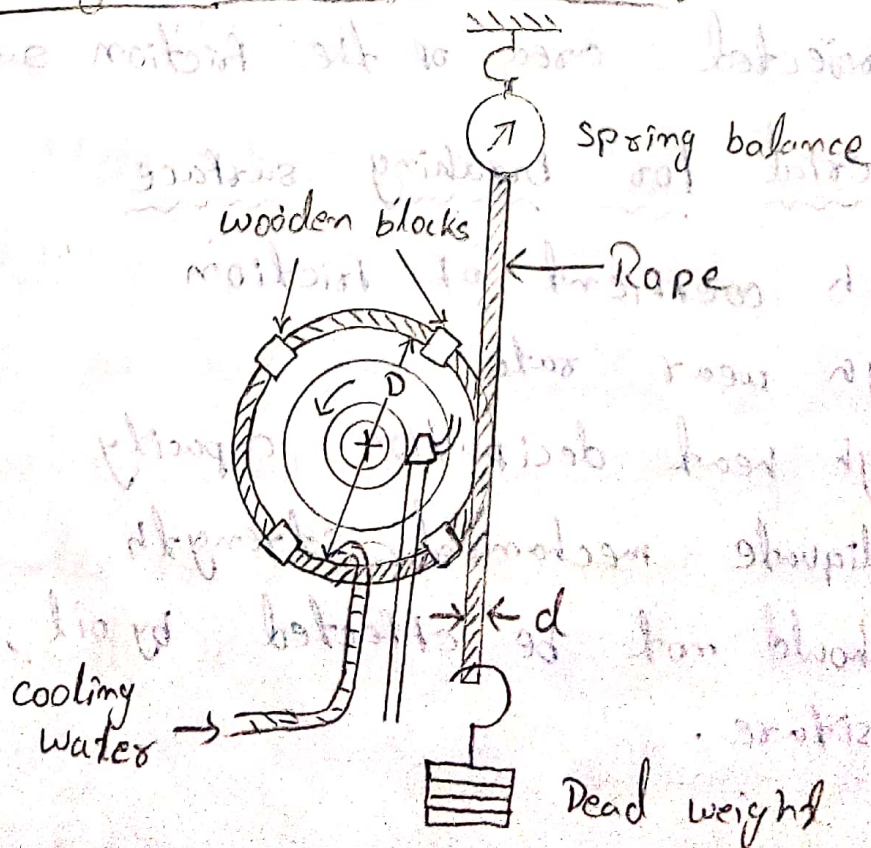
Dynamometer

absorption dynamometer

transmission dynamometers



Prony brake dynamometer



Rope brake dynamometer

Prony brake dynamometer

w = weight at the outer end of the lever in newtons

L = Length between w and center of the pulley

$$T = w \cdot L = F \cdot R \quad \text{N-m}$$

Break power

$$B.P = \frac{2\pi N T}{60} = \frac{w \cdot L \times 2\pi N}{60} \quad \text{watt}$$

Rope brake dynamometer

$$B.P = \frac{(w-s) \pi D N}{60} \quad \text{watt}$$

$$\text{or} \\ B.P = \frac{(w-s) \pi (D+d) N}{60} \quad \text{watt}$$

Dt 22.01.2020

Q In an experiment the following observations are obtained.

Dead weight = 600 N

Spring load = 180 N

r.p.m = 200

Dia. of pulley = 1.2 m

dia. of rope = 12.5 mm

Find the brake power obtained

$$B.P = \frac{(w-s) \pi (D+d) N}{60} \\ = \frac{(600 - 180) \pi (1.2 + 0.0125) 200}{60} = 5332.85 \quad \text{W}$$

Chapter - 3

Power Transmission

Flat Belt Drive power Transmission

- ① Belt drive
 - ② Chain drive
 - ③ Rope drive
 - ④ Gear drive
- └ Flat belt drive
└ V-belt drive

Types of belt drive as per speed of belt

$v \leq 10 \text{ m/s}$ ① Low velocity drive

$v > 10 \text{ m/s}$ ② medium velocity drive
 $< 22 \text{ m/s}$

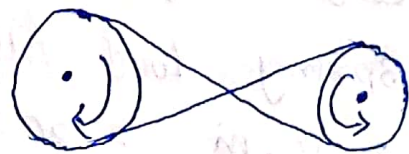
$v > 22 \text{ m/s}$ ③ High velocity drive

Types of Flat belt drive according to connection of belt

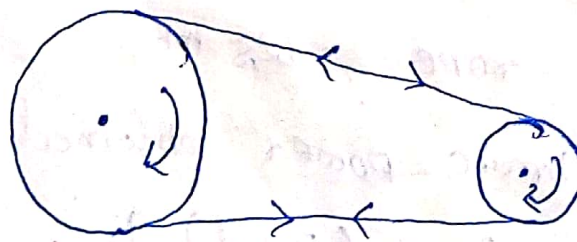
① open belt drive

② Crossed belt drive

③ Quarter turn belt drive



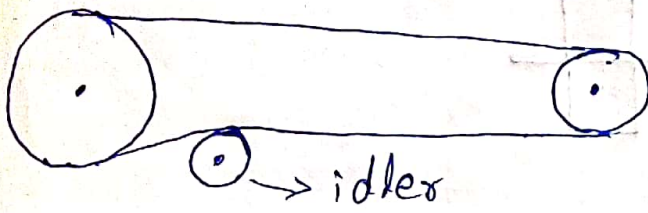
Cross belt drive



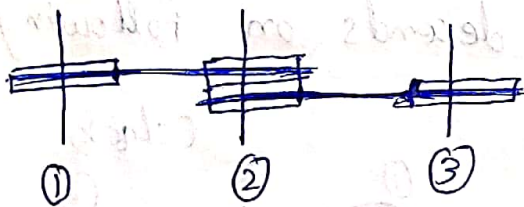
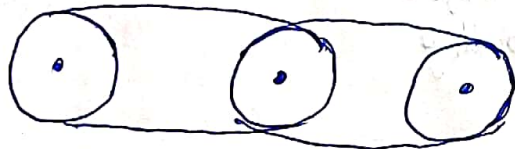
Driver

Driven
(follower)

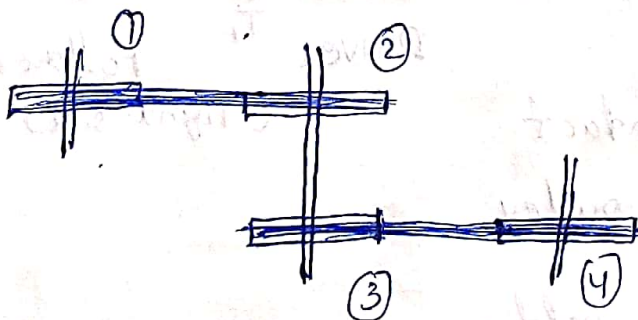
④ Belt drive with idler pulley



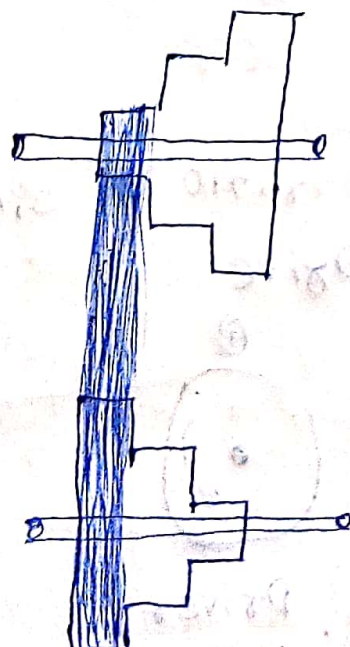
⑤ Simple belt drive



⑥ Compound belt drive



⑦ Stepped or Cone pulley drive

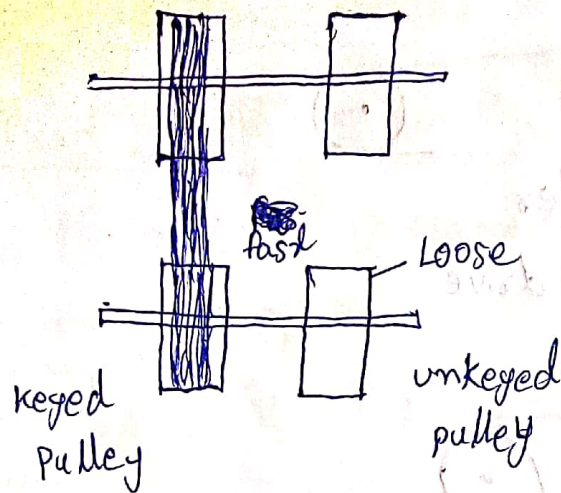


Drivers

Driven

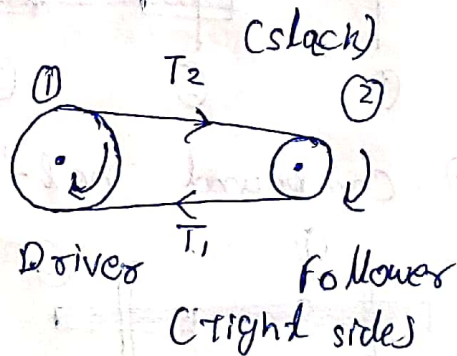
$$V = \frac{\pi D N}{60}$$

⑧ Fast and loose pulley drive



Power transmission depends on following factors

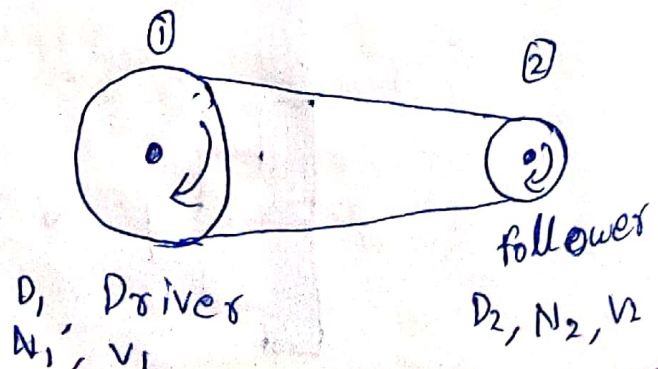
- ① velocity or belt (v).
- ② Arc of contact of smaller pulley.
- ③ Tension in belt.



$$P = (T_1 - T_2) v$$

Derive the velocity ratio of simple and compound belt drive

In one revolution
distance moved by
pulley ① = πD_1



In N_1 revolution distance = $\pi D_1 N_1$ m/min

$$V_1 = \frac{\pi D_1 N_1}{60} \text{ m/s}$$

$$V_2 = \frac{\pi D_2 N_2}{60} \text{ m/s}$$

$$V_1 = V_2$$

$$\frac{\pi D_1 N_1}{60} = \frac{\pi D_2 N_2}{60}$$

$$\boxed{\frac{N_2}{N_1} = \frac{D_1}{D_2}}$$

If thickness of belt is considered (t)

$$\boxed{\frac{N_2}{N_1} = \frac{D_1 + t}{D_2 + t}}$$

If there is slip

$$\boxed{\frac{N_2}{N_1} = \frac{D_1}{D_2} \left(1 - \frac{S}{100}\right)}$$

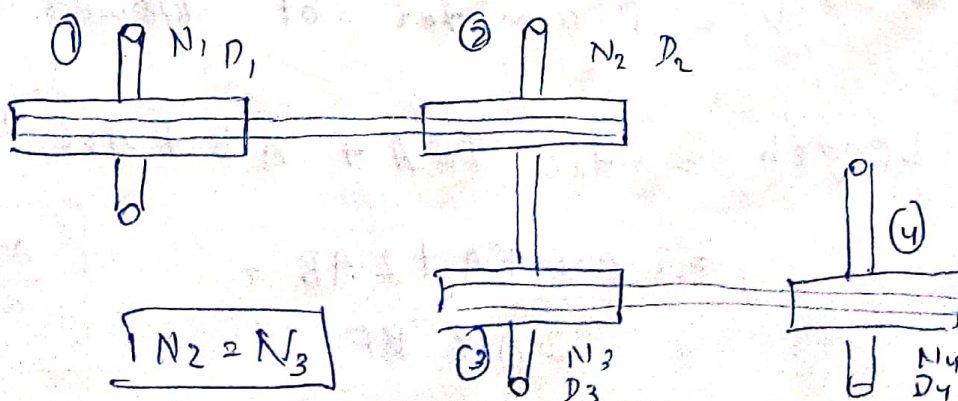
$$S = S_1 + S_2$$

S_1 = slip % in Drives

S_2 = slip % in follower

dt 27.1.2020

Velocity ratio of compound belt drive



$$\frac{N_2}{N_1} = \frac{D_1}{D_2} \quad , \quad \frac{N_4}{N_3} = \frac{D_3}{D_4}$$

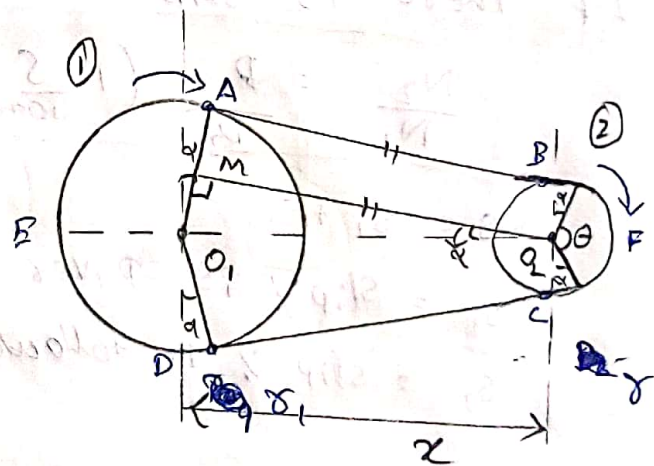
$$\frac{\cancel{N_2}}{N_1} \times \frac{N_4}{\cancel{N_3}} = \frac{D_1 D_3}{D_2 D_4}$$

$$\frac{N_4}{N_1} = \frac{D_1 D_3}{D_2 D_4}$$

r.p.m of last follower
r.p.m of 1st driver

= $\frac{\text{product of diameter of drivers}}{\text{product of diameter of followers}}$

Derive the formula for length of belt for open belt drive



Let D_1 = Diameter of Driver
 D_2 = Diameter of follower

$$\begin{aligned} \text{Length} &= \text{Arc DBA} + AB + \text{Arc BFC} + CD \\ &= 2 \text{Arc EA} + 2AB + 2 \text{Arc BF} \end{aligned}$$

$$\boxed{\frac{\text{Arc}}{\text{radius}} = \text{angle}}$$

$$= 2 (\arccos BA + AB + \arccos BP)$$

$$= 2 \left[\left(\frac{\pi}{2} + \alpha \right) r_1 + \sqrt{x^2 - (r_1 - r_2)^2} + \left(\frac{\pi}{2} - \alpha \right) r_2 \right]$$

construct a line m O_2 parallel to the line AB . Angle between $\angle O_1 O_2 m = \alpha$

$$O_1 O_2 = x$$

$$O_1 m = r_1 - r_2$$

$$O_2 m = AB = \sqrt{x^2 - (r_1 - r_2)^2}$$

$$L_{\text{length}} = 2 \left[r_1 \cdot \frac{\pi}{2} + r_1 \alpha + x \sqrt{1 - \left(\frac{r_1 - r_2}{x} \right)^2} + r_2 \cdot \frac{\pi}{2} - r_2 \alpha \right]$$

we know $\sin \alpha = \frac{r_1 - r_2}{x}$ or

$$\alpha = \frac{r_1 - r_2}{x} \quad \text{if } \alpha \text{ is small}$$

$$L = 2 \left[r_1 \cdot \frac{\pi}{2} + r_1 \frac{(r_1 - r_2)}{x} + x \sqrt{1 - \left(\frac{r_1 - r_2}{x} \right)^2} + r_2 \cdot \frac{\pi}{2} - r_2 \frac{(r_1 - r_2)}{x} \right]$$

$$L = 2 \left[r_1 \cdot \frac{\pi}{2} + r_1 \frac{(r_1 - r_2)}{x} + x \left(1 - \frac{1}{2} \cdot \left(\frac{r_1 - r_2}{x} \right)^2 \right) + r_2 \cdot \frac{\pi}{2} - r_2 \frac{(r_1 - r_2)}{x} \right]$$

$$L = 2 \left[\frac{\pi}{2} (r_1 + r_2) + \frac{(r_1 - r_2)}{x} (r_1 - r_2) + x - \frac{(r_1 - r_2)^2}{2x} \right]$$

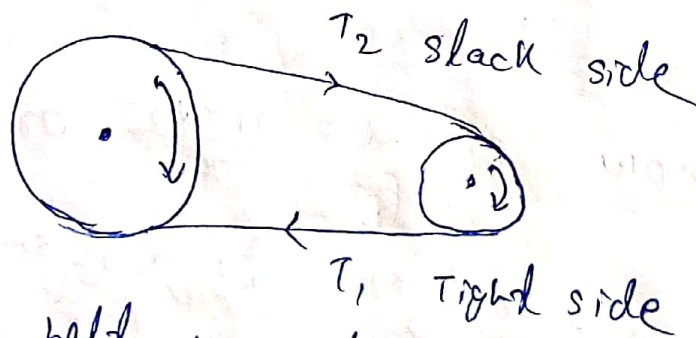
$$= \pi (r_1 + r_2) + \frac{2}{x} (r_1 - r_2)^2 + 2x - \frac{(r_1 - r_2)^2}{2x}$$

$$L = \pi (r_1 + r_2) + \frac{(r_1 - r_2)^2}{x} + 2x$$

Assignment

Derive the formula for length of belt for cross belt drive.

Creep of Belt



As the belt is continuous by moving from slack side to tight side the belt is expanded and when the belt is come to tight side to slack side then it contraction

If creep occurs

$$\frac{N_2}{N_1} = \frac{D_1}{D_2} \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$$

E = Young's modulus

σ_1 = stress of belt in tight side

σ_2 = stress of belt in slack side

Q Derive the ratio of driving tension for flat belt drive or prove that

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

θ = Angle of contact

μ = coefficient of friction between belt and the pulley

$$\log_e \frac{T_1}{T_2} = \mu\theta$$

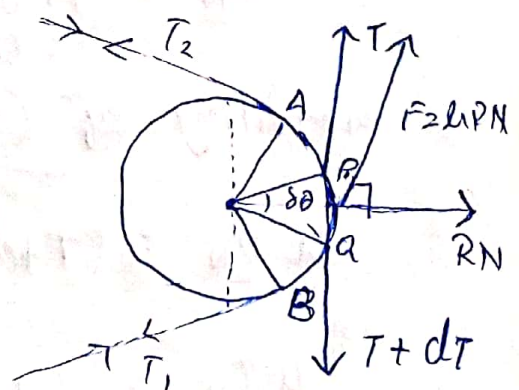
Considering equilibrium
resolving the forces

$$\sum H = 0$$

$$\Rightarrow T \sin \frac{\delta\theta}{2} + (T + dT) \sin \frac{\delta\theta}{2} = R_N \quad \text{--- (1)}$$

$$\sum V = 0$$

$$\Rightarrow T \cos \frac{\delta\theta}{2} + \mu R_N = (T + dT) \cos \frac{\delta\theta}{2} \quad \text{--- (2)}$$



Putting $\sin \frac{\delta\theta}{2} = \frac{\delta\theta}{2}$

$\cos \frac{\delta\theta}{2} = 1$

solving eqⁿ ① and ②

we get

$\log e \frac{T_1}{T_2} = \mu\theta$

$\Rightarrow \frac{T_1}{T_2} = e^{\mu\theta}$

simplifying eqⁿ ①,

$R_N = (T + dT) \frac{\delta\theta}{2} + T \times \frac{\delta\theta}{2} \left\{ \sin \frac{\delta\theta}{2} = \frac{\delta\theta}{2} \right\}$

$= \frac{T\delta\theta}{2} + \frac{dT\delta\theta}{2} + \frac{T\delta\theta}{2}$

$= \frac{2T\delta\theta}{2}$

$\left[\frac{dT\delta\theta}{2} \text{ is very small} \right]$

$R_N = T \cdot \delta\theta \quad \text{--- ③}$

simplifying eqⁿ ②

$\mu R_N = (T + dT) \cos \frac{\delta\theta}{2} - T \cos \frac{\delta\theta}{2}$

$\mu R_N = T + dT - T$

$\mu R_N = dT$

$\left[\cos \frac{\delta\theta}{2} = 1 \right]$

$R_N = \frac{dT}{\mu}$

--- ④

Putting the value of R_N in eqn (4)

$$T \delta \theta = \frac{dT}{\mu}$$

$$\frac{dT}{T} = \frac{\delta \theta}{\mu} \times \mu$$

Integrating both sides by T_2 and T_1 and from 0 to θ

$$\int_{T_2}^{T_1} \frac{dT}{T} = \mu \int_0^\theta \delta \theta$$

$$\log_e \left[\frac{T_1}{T_2} \right] = \mu \theta$$

$$\boxed{\frac{T_1}{T_2} = e^{\mu \theta}}$$

Dt 28.01.2020

Power transmitted by flat belt drive

$$\boxed{P = (T_1 - T_2) v}$$

Condition for maximum power

Q. What is the condition for maximum power transmitted by a flat belt drive

$$\boxed{T = 3 T_c}$$

$$\boxed{T = T_1 + T_c}$$

T = maximum tension

T_c = Centrifugal tension

$$\boxed{T_c = mv^2}$$

$$P = (T_1 - T_2) v$$

$$= \left(T_1 - \frac{T_1}{\gamma} \right) v$$

$$= T_1 \left(1 - \frac{1}{\gamma} \right) v$$

$$= T_1 \cdot v \cdot \gamma$$

$$= (T - T_c) v \cdot c$$

$$= T \cdot v \cdot c - mv^2 \cdot v \cdot c$$

$$= T \cdot v \cdot c - mv^3 \cdot c$$

For maximum power

$$\frac{dp}{dv} = 0$$

$$\frac{dp}{dv} = T \cdot c - mv^2 \cdot c$$

$$0 = T - m3v^2$$

$$T = m3v^2$$

$$\boxed{T = 3T_c}$$

$$\boxed{v = \sqrt{\frac{T}{3m}}}$$

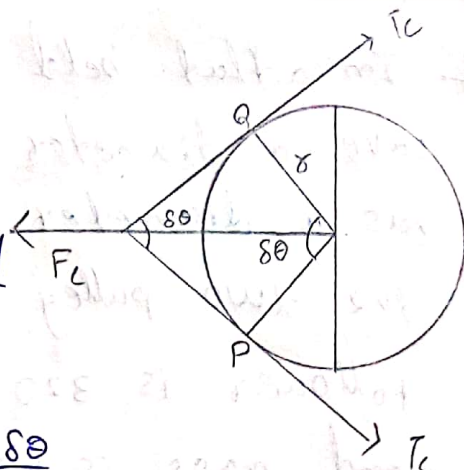
$$\boxed{P_{\max} = (T_1 - T_2) \sqrt{\frac{T}{3m}}}$$

Prove that $T_c = mv^2$

Let m = mass of belt per unit length

Then length of PQ
 $= m \cdot r \cdot \delta \theta$

Resolving the force horizontal
 and vertical force



$$F_c = T_c \cos \frac{\delta \theta}{2} + T_c \cos \frac{\delta \theta}{2}$$

$$F_c = 2 T_c \cos \frac{\delta \theta}{2}$$

$$\frac{m v^2}{r} = 2 T_c$$

$$T_c = \frac{m v^2}{2 r}$$

$$= \frac{m \cdot r \cdot d\theta}{2 r}$$

Initial tension

$$T_0 = \frac{T_1 + T_2 + 2 T_c}{2}$$

$$= \frac{T_1 + T_2}{2}$$

* If $v > 10 \text{ m/s}$ T_c is considered

1Q] In a belt drive the belt is 1 kg/m length and velocity is 12 m/sec and it transmits power of 9.6 kW if coefficient of friction is 0.3 and angle of contact is 205° determine initial tension and strength of the belt.

2Q In a flat belt drive the driver pulley have a diameter of 1.52 m and driven pulley has a diameter of 0.8 m and distance between the two pulley is 4 m. RPM of the follower is 320. Initial tension is 2.8 kN and mass is 1.8 kg/m. If coefficient of friction is 0.25 find the power.

3Q Two parallel shafts on apparatus are provided with 300 mm and 400 mm diameter pulley and are connected by means of cross belt. The direction of rotation of the follower is to be reversed by changing over to an open belt drive. How much length of the belt has to be reduced.

Dt 29.01.2020

$$(1) L_o = \pi (r_1 + r_2) + \frac{(r_1 - r_2)^2}{x} + 2x$$

$$(2) L_c = \pi (r_1 + r_2) + \frac{(r_1 + r_2)^2}{x} + 2x$$

$$(3) P = (T_1 - T_2) v$$

$$(4) \frac{T_1}{T_2} = e^{\mu \theta}$$

$$(5) \sin \alpha = \frac{r_1 - r_2}{x} \quad \text{or} \quad \frac{r_1 + r_2}{x}$$

$$(6) \theta = (180^\circ - 2\alpha) \frac{\pi}{180} \quad \text{or} \quad (180^\circ + 2\alpha) \frac{\pi}{180}$$

$$(7) T = 3T_c \quad \text{-(for max power)}$$

$$(8) \quad v = \sqrt{\frac{T}{3m}} \quad (\text{For max}^m \text{ power})$$

$$(9) \quad T_c = m v^2$$

$$(10) \quad T = \text{max}^m \text{ tension} = T_1 + T_c$$

$$(11) \quad T = \text{max}^m \text{ tension} = \sigma \cdot b \cdot t$$

where σ = stress of belt

b = width of belt

t = thickness of belt

$$(12) \quad m = \rho \cdot l \cdot b \cdot t$$

l = length of belt

ρ = density of belt

$$(13) \quad T_0 = \frac{T_1 + T_2 + 2T_c}{2} \quad \text{or} \quad \frac{T_1 + T_2}{2}$$

$$(14) \quad \frac{N_2}{N_1} = \frac{d_1}{d_2}$$

$$(15) \quad \frac{N_2}{N_1} = \left(\frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}} \right)$$

$$(16) \quad \frac{N_2}{N_1} = \frac{d_1}{d_2} \left(1 - \frac{s}{100} \right)$$

$$(17) \quad \frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}$$

Q4 A leather belt 125 mm width and 6 mm thick and transmit power from pulley of 750 mm diameter running at 450 rpm if angle of contact is 140° and $\mu = 0.3$ and mass of 1 m^3 of leather belt is 1100 kg and stress

of the belt 2.75 mpa . find maximum power

⑤ Find the power required for a belt drive running at velocity of 600 m/min where coefficient of friction is 0.3 angle of contact 160° and maximum tension 700 N

⑥ Find the width of the belt necessary for a flat belt drive transmitting power of 7.5 kW from a pulley of diameter 300 mm at a rpm of 600 , coefficient of friction 0.22 , angle of contact 200° and maximum tension is 8 N/m width of the belt.
Assume $t = 10 \text{ mm}$.

⑦ Write the advantage and disadvantage of v belt drive over flat belt drive.

Gear / gear train

Dt 3.02.2020

Addendum circle

Dedendum circle

pitch circle

Addendum $\rightarrow$ The distance or height from the pitch circle to addendum circle

Dedendum $\rightarrow$ The distance or height from the pitch circle to dedendum circle

module (m) :- $\left(\frac{D}{T}\right)$

It is the ratio between pitch circle diameter to the number of teeth

Diametrical pitch $P_d = \frac{T}{D} = \frac{1}{m}$

reciprocal of module (m)

clearance

~~then~~ The distance between the bottom of one tooth to the top of the other tooth is called clearance.

Difference types of gear

$\rightarrow$ ① according to position of the shaft of the gear

② Parallel shaft gears.

Ex spur gear

② Non parallel intersecting

Ex Bevel gear

③ Non parallel non intersecting (Non coplanar)

Ex spiral gear

③ According to speed of the gear

① Low velocity gear $v > 3.5$ ~~m/s~~ m/s

② medium velocity gear $3.5 \geq 10$ ~~m/s~~ m/s

③ high velocity gear < 10 ~~m/s~~ m/s

④ According to arrangement

① External

② Internal

⑤ According to position of the teeth of the gear

① straight tooth gear

② Inclined tooth gear

③ Rack and pinion

10.02.2020

In a flat belt drive the driver pulley have a diameter of 1.5 m and driven pulley is 4m dpm of the following is 320. ~~Internal~~
Initial tension is 2.8 kN and mass is 1.8 kg/m²
coefficient of friction is 0.25 find the power

$$d_1 = 1.5 \text{ m}$$

$$d_2 = 0.8 \text{ m}$$

$$x = 4 \text{ m}$$

$$N_1 = 320 \text{ rpm}$$

$$T_0 = 2.8 \text{ kN}$$

$$m = 1.8 \text{ kg/m}$$

$$\mu = 0.25$$

Formulas used

$$P = (T_1 - T_2) V$$

$$V = \frac{\pi D_2 N}{60}$$

$$\frac{T_1}{T_2} = e^{\mu \theta}$$

$$T_0 = \frac{T_1 + T_2 + 2T_L}{2}$$

$$T_L = mv^2$$

$$\theta = (180 - 2\alpha) \frac{\pi}{180}$$

$$\sin d = \frac{\gamma_1 - \gamma_2}{\pi}$$

$$V = \frac{\pi D_2 N}{60}$$

$$= \frac{\pi \times 0.8 \times 320}{60}$$

$$= 13.4041 \text{ m/s}$$

$$\sin d = \frac{0.75 - 0.4}{\pi}$$

$$= 0.0875$$

$$d = \sin^{-1} 0.0875$$

$$= 5.0198$$

$$\theta = (180 - 2 \times 5.0198) \frac{\pi}{180}$$

$$= 2.9663$$

$$T_c = mv^2$$

$$= 1.8 \times (13.4041)^2$$

$$= 323.4058 \text{ N}$$

$$\frac{T_1}{T_2} = e^{0.25 \times 2.9663}$$

$$\frac{T_1}{T_2} = 2.099$$

$$T_1 = 2.099 T_2$$

$$T_0 = \frac{T_1 + T_2 + 2T_c}{2}$$

$$2.8 \times 10^3 = \frac{2.099 T_2 + T_2 + 2 \times 323.4058}{2}$$

$$5.6 \times 10^3 = 3.099 T_2 + 646.8116$$

$$3.099 T_2 = 5.6 \times 10^3 - 646.8116$$

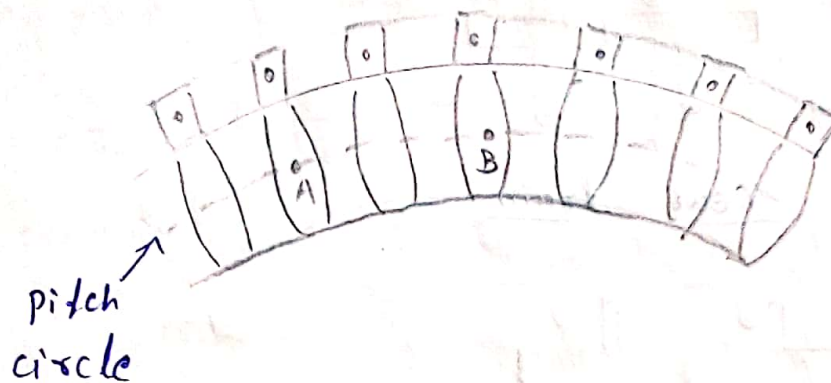
$$T_2 = \frac{4953.1884}{3.099}$$

$$= 1598.31 \text{ N}$$

$$T_1 = 2.099 \times 1598.31$$

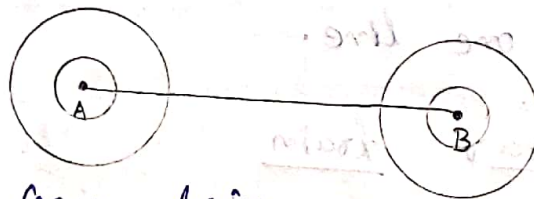
$$= 3354.85 \text{ N}$$

$$\text{Circular pitch } (P_c) = \frac{\pi D}{T}$$



It is the distance measured along pitch circle circumference from a point of 1 tooth to the corresponding point of next adjacent tooth

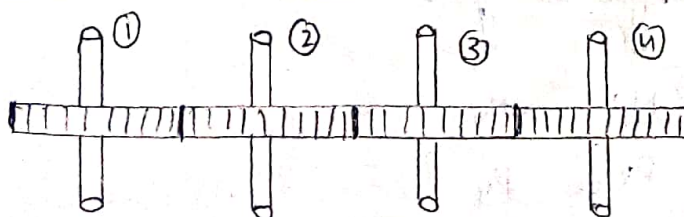
Gear train



Types of gear train

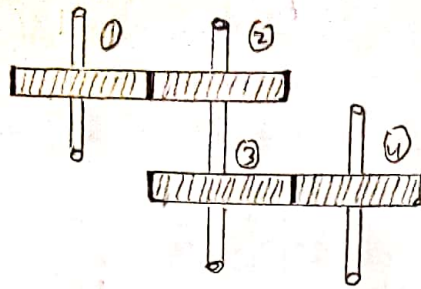
- ① Simple gear train
- ② Compound gear train
- ③ Reversed gear train
- ④ Epicyclic gear train

Simple gear train

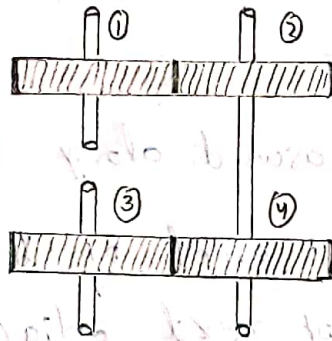


one shaft one gear

Compound Gear Train

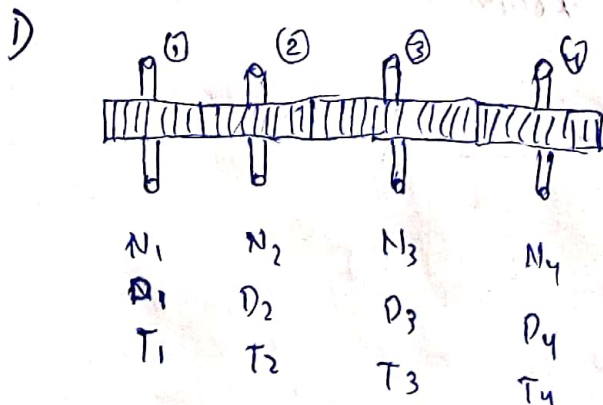
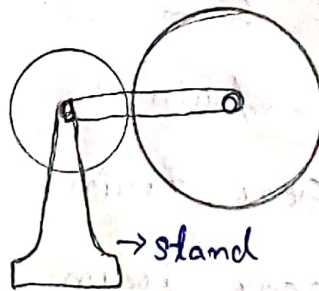


Reversed Gear Train



It is the axis of the shaft driven and axis of the shaft of the last follower remain in one line.

Epicyclic Gear Train



Consider gears ① and ②

$$\frac{N_2}{N_1} = \frac{T_1}{T_2} = \frac{D_1}{D_2} \quad \text{--- ①}$$

consider ③ and ④

$$\frac{N_4}{N_3} = \frac{T_3}{T_4} \quad \text{--- ②}$$

consider ② and ③

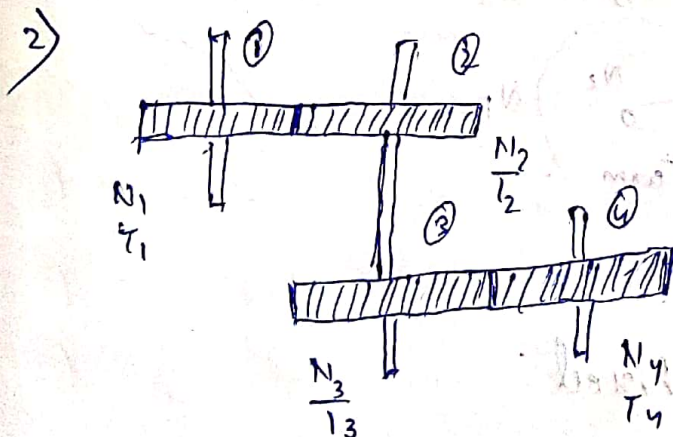
$$\frac{N_3}{N_2} = \frac{T_2}{T_3} \quad \text{--- ③}$$

multiplying equation ① ② and ③

$$\frac{\cancel{N_2}}{N_1} \times \frac{N_4}{\cancel{N_3}} \times \frac{\cancel{N_3}}{\cancel{N_2}} = \frac{T_1}{T_2} \times \frac{T_3}{T_4} \times \frac{T_2}{T_3}$$

$$\boxed{\frac{N_4}{N_1} = \frac{T_1}{T_4}}$$

~~d.p.m of last follower~~
d.p.m of first driver



Considering gears ① and ②

$$\frac{N_2}{N_1} = \frac{T_1}{T_2} \quad \text{--- ①}$$

Considering ③ and ④

$$\frac{N_4}{N_3} = \frac{T_3}{T_4} \quad \text{--- ②}$$

multiply my equation ① and ②

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} = \frac{T_1 T_3}{T_2 T_4}$$

$$\frac{N_4}{N_1} = \frac{T_1 T_3}{T_2 T_4}$$

Product of no of teeth of drivers

Product of no of teeth of followers

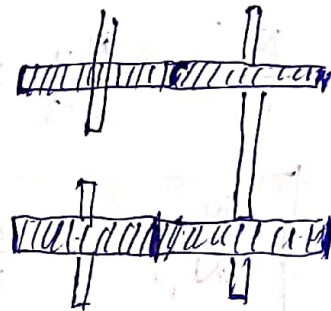
③ Reversed

$$\frac{N_4}{N_1} = \frac{T_1 T_3}{T_2 T_4}$$

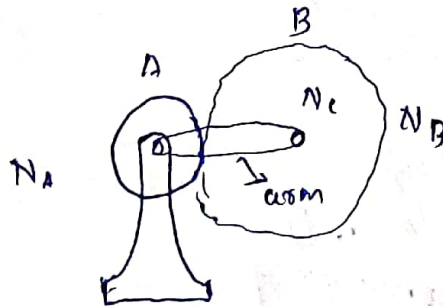
Condition

$$r_1 + r_2 = r_3 + r_4$$

$$T_1 + T_2 = T_3 + T_4$$



11.02.2020



If N_C is fixed

$$\frac{N_B}{N_A} = \frac{T_A}{T_B}$$

$$\boxed{\frac{N_B - N_C}{N_A - N_C} = \frac{T_A}{T_B}}$$

In algebraic method of calculation velocity ratio formula

$$\frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B}$$

Tabular method

Step No.	Condition of motion	Revolution of Element		
		Arm 'c'	Gear A	Gear B
1	Gear A rotates +1 revolution clockwise	0	+1	$-\frac{T_A}{T_B}$
2	Gear A rotates 'x' revolution c.w [multiply by cn]	0	+x	$x \times \frac{T_A}{T_B}$
3	Gear A rotates 'x' revolution and add 'y' revolution	$0+y=y$	$x+y$	$y - x \frac{T_A}{T_B}$
4	Total motion	y	$x+y$	$y - x \frac{T_A}{T_B}$

Q. In an epicyclic gear train, an arm carries two gears A and B having 36 and 40 teeth respectively. If the arm rotates at 150 rpm in an anticlockwise direction about the center of gear A which is fixed. Determine the speed of gear B - if the gear A is in a state of being fixed makes 300 rpm in the clockwise direction which will be the speed of the gear B.

$$y = -150$$

$$x + y = 0$$

$$x = 150$$

$$N_B = y - x \frac{T_A}{T_B}$$

$$N_B = -150 - 150 \frac{36}{40}$$

$$= -270$$

$$y = -150$$

$$x + y = 300$$

$$x - 150 = 300$$

$$x = 450$$

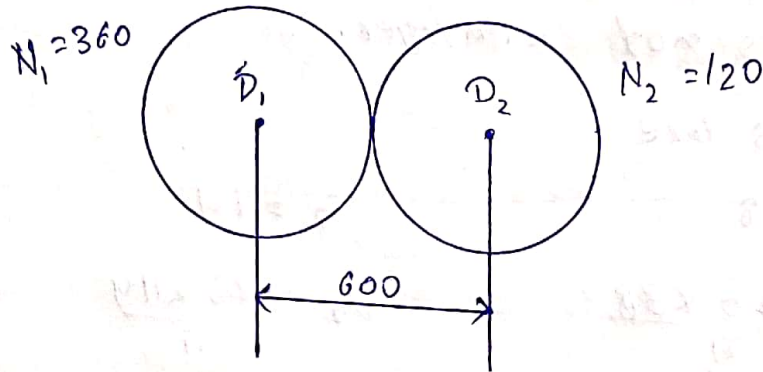
$$N_B = y - x \frac{T_A}{T_B}$$

$$= -150 - 450 \times \frac{36}{40}$$

$$= -510$$

$$\text{module } (m) = \frac{D}{T}$$

$$\text{circular pitch } p_c = \frac{\pi D}{T}$$



$$\frac{D_1}{2} + \frac{D_2}{2} = 600$$

$$R_1 + R_2 = 600$$

$$D_1 + D_2 = 1200$$

$$\frac{N_2}{N_1} = \frac{T_1}{T_2}$$

$$\frac{120}{360} = \frac{T_1}{T_2} \quad \text{--- (2)}$$

$$\frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} = p_c$$

$$\frac{D_1}{D_2} = \frac{T_1}{T_2} = \frac{1}{3}$$

$$D_1 = \frac{1}{3} D_2$$

$$D_1 + 3 D_1 = 1200$$

$$D_1 = \frac{1200}{4} = 300 \text{ mm}$$

$$D_2 = 900$$

$$\begin{aligned} \frac{\pi D_1}{T_1} &= 25 \Rightarrow T_1 = \frac{\pi D_1}{25} \\ &= \frac{\pi \times 300}{25} \\ &= 37.69 \text{ N} \end{aligned}$$

$$\frac{\pi D_2}{T_2} = 25$$

$$T_2 = \frac{\pi D_2}{25}$$

$$= \frac{\pi \times 900}{25} = 113.09$$

From the problem the number of teeth or gear 1 is not an integer.

Let us take

$$T_1 = 38$$

$$T_2 = 114$$

$$D_1 = \frac{25 \times 38}{\pi}$$

$$D_2 = \frac{25 \times 114}{\pi}$$

$$= 302.39 \text{ mm}$$

$$= 907.18 \text{ mm}$$

$$r_1 = \frac{302.39}{2}$$

$$= 151.17$$

$$r_2 = \frac{907.18}{2}$$

$$= 453.59$$

$$r_1 + r_2 = 604.76 \text{ mm.}$$

GOVERNOR

Two thousand 10

02

Function - 1) It regulates the mean speed of an engine when there are variations of loads.

WEDNESDAY June

2) When load on the engine increases speed decreases so it becomes necessary to ^{increase} supply of working fluid.

3) When load on the engine decreases speed increases so less working fluid is reqd.

4) The governor automatically controls the supply of working fluid under variations of load and keeps the mean speed within certain limits.

Working:- When load increases, the configuration of governor changes and a valve is moved to increase the supply of the working fluid. Conversely when the load decreases, the speed of engine increases and governor decreases the supply of working fluid.

CLASSIFICATION. ^① CENTRIFUGAL ^② INERTIA

Pendulum type

(Watt Governor)

loaded type

Dead weight type

Loaded type

Porter

Proell

Hartnell

Wilson

Hartnell

Governor with gravity Spring Control

Hartung Pickering

JUN '10

01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

Centrifugal Governor - These governors are based on the balancing of centrifugal force on the rotating balls by an equal and opposite radial force known as controlling force (acting gravity or spring).

The shaft of a prime mover is connected to the governor shaft by means of gear or belt drive. Modern governors get their connection from the prime mover shaft through some servomechanism or electronic device. In a centrifugal governor two balls are fixed ^{through} the arms to the shaft of governor. The balls thus revolve with the shaft giving rise to centrifugal force acting radially outwards. The centrifugal force is balanced by controlling force acting radially inwards which is provided by a dead weight, spring or both. The governor balls or fly balls revolve with a spindle which is driven by the engine through bevel gears. The upper ends of arms are pivoted to the spindle so that the balls may rise up or fall down as they revolve about the vertical axis. The arms are connected by the links to a sleeve which is keyed to the spindle. This sleeve revolve with spindle but can slide up and down. The ball and the sleeve rises when spindle speed increases and falls when speed decreases.

04

FRIDAY June

In order to limit the travel of sleeve in upward and downward directions, two stops S.S are provided on spindle. The sleeve is connected by a bell crank lever to a throttle valve. The supply of working fluid decreases when sleeve rises and increases when it falls.

When the load on the engine increases the engine and the governor speed decreases. This reduces centrifugal force on balls. So balls move inwards and sleeve moves downwards. The downward movement of sleeve operates a throttle valve at the other end of the bell crank lever to increase the supply of working fluid and thus engine speed is increased. In this case the extra power output is provided to balance the increased load. When the load on the engine decreases, the engine and the governor speed increases, which results in the increase of centrifugal force on the balls. Thus the balls move outwards and the sleeve rises upwards. This upward movement of the sleeve reduces the supply of the working fluid and hence the speed is decreased. In this case the power output is reduced.

Note: When the balls rotate at uniform speed, Centrifugal force = Centrifugal force. So they balance each other.

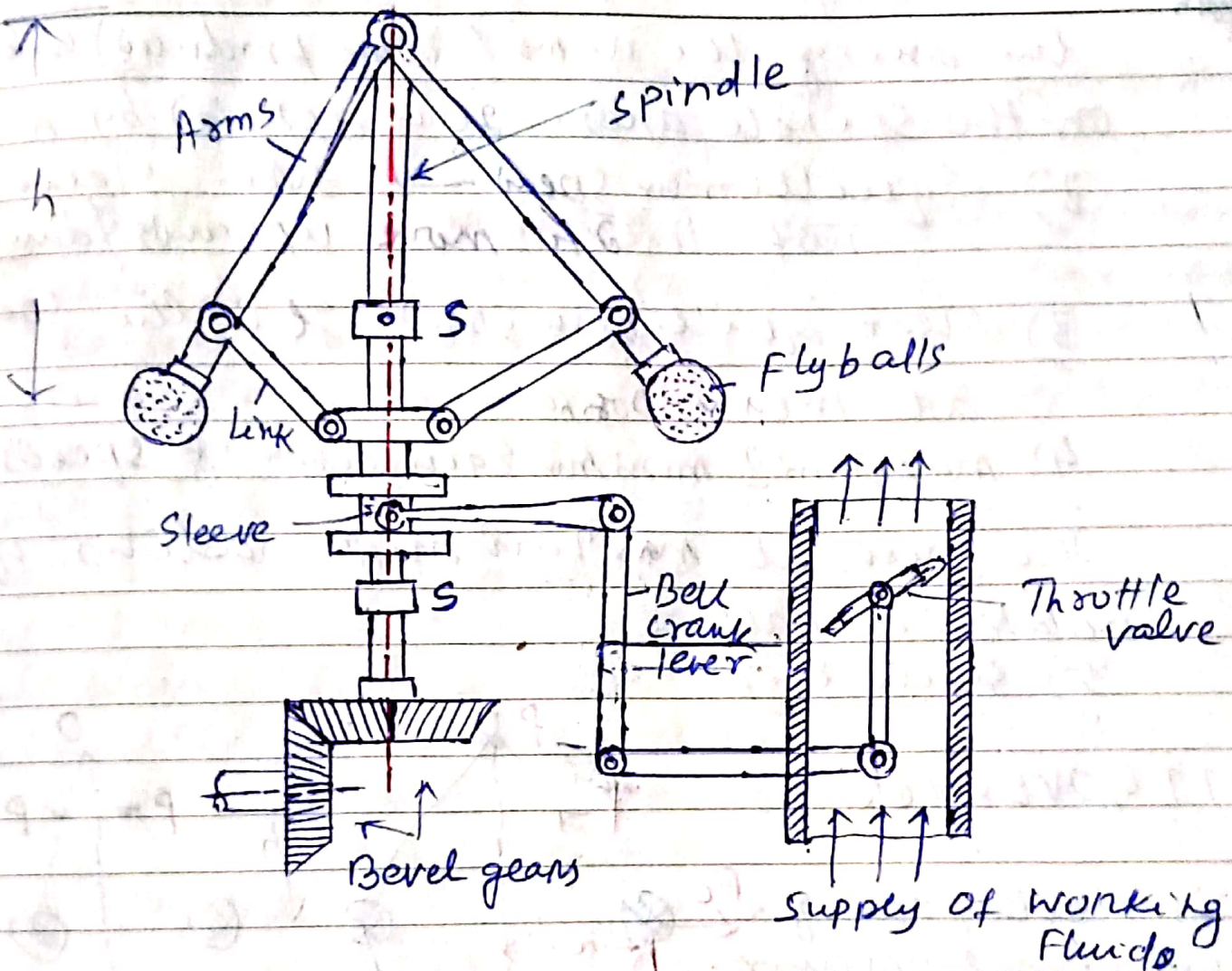
JUN '10 01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

Centrifugal Governor.

June SATURDAY

05

156-209 / 23rd wk



Inertia governor. In this case the positions of the balls are effected by the rate of change of speed i.e. angular acceleration or retardation of governor shaft.

Since the action of governor is due to rate of change of speed and not a finite change of speed, a more rapid response to change of load is obtained. However due to practical difficulty of balancing the inertia force caused by balls of the governor to the controlling force, this type of governor is not preferred to centrifugal type.

TERMS USED IN GOVERNORS.

Two thousand 10

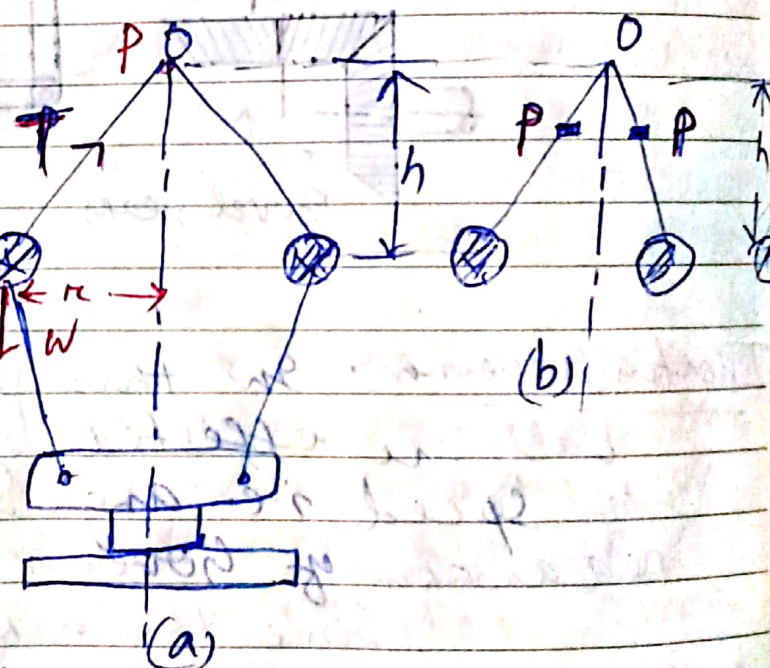
07

MONDAY June

- 1) Height of a governor - (h) - vertical distance from centre of ball to a point where the axes of the arms (arms produced) intersect on the spindle axis. It is denoted by h.
- 2) Equilibrium speed $\rightarrow$ when sleeve does not tend to move upwards/downwards.
- 3) Mean Equilibrium speed - It is the speed at mean position.
- 4) Maximum & minimum Equilibrium of speeds - The speeds at max^m and min^m radius of rotation of balls.
- 5) Sleeve lift

WATT GOVERNOR

Simplest form of a centrifugal governor. It is basically a conical pendulum with links attached to a sleeve of negligible mass.



Let m = mass of ball in kg.

W = Wt. of ball in N = mg .

T = Tension in the arm in N

ω = angular vel. of arm & ball about spindle axis

r = radius

F_c = centrifugal force = $m\omega^2 r$

JUN'10

01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

$$F_c \cdot h = (W \cdot r) = m g r, F_c = m \omega^2 r.$$

$$\Rightarrow h = \frac{2}{\omega^2}$$

$$\omega = 2\pi N/60$$

$$h = \frac{9.81}{(2\pi N/60)^2} = \frac{895}{V^2} \text{ m (60 to 80 rpm)}$$

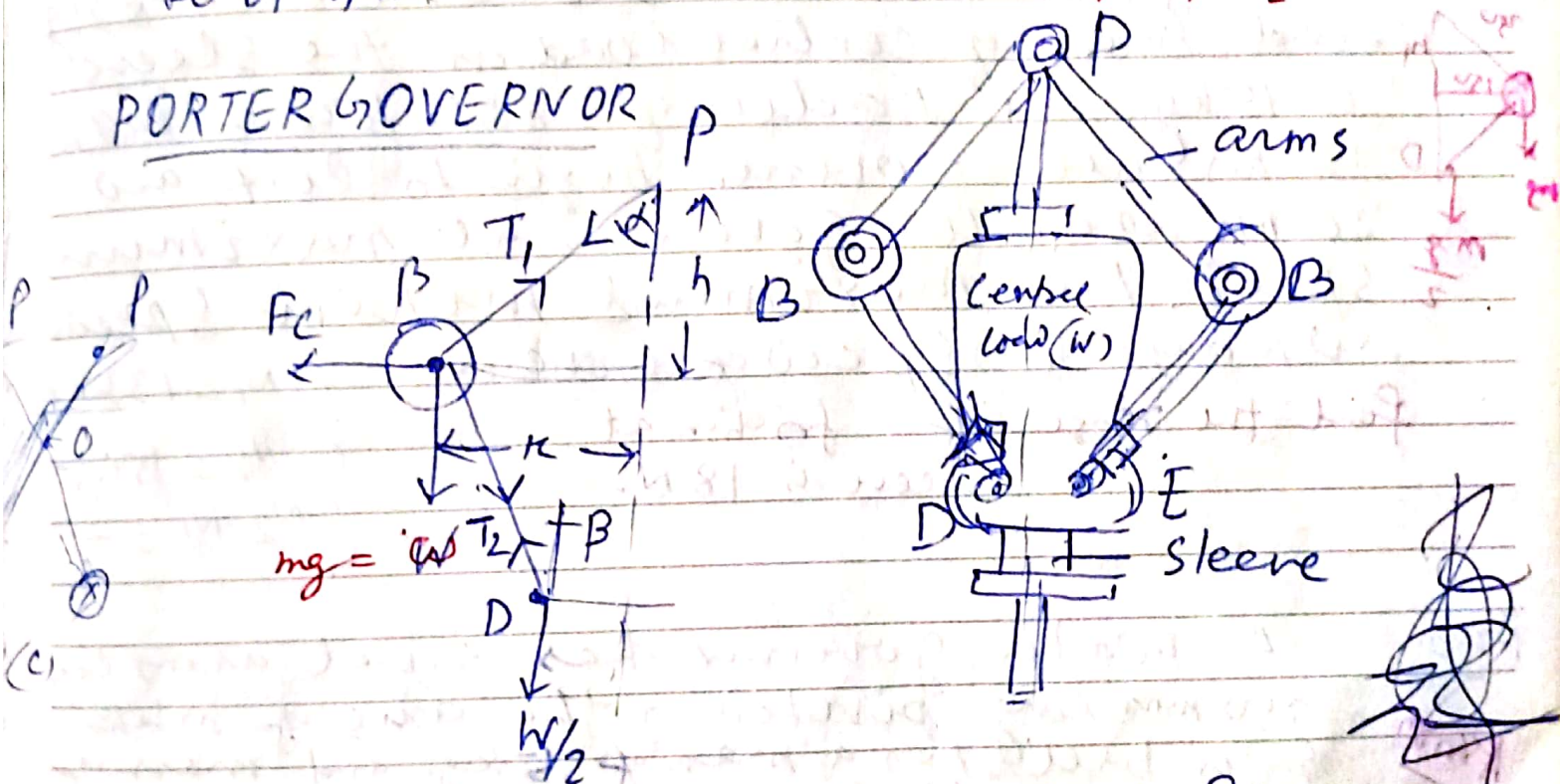
June 1 TUESDAY

08

Prob. Calculate the vertical ht. of a Watt governor when it rotates at 60 R.P.M. Also find the change in vertical height when its speed increases to 81 R.P.M. $h = h_1 - h_2 = 0.008 \text{ m}$

$$h = h_1 - h_2 = 0.008 \text{ m}$$

PORTER GOVERNOR



$m \pm$ many each ball

$W = \text{wt. of each ball} = mg$

$M = \text{mean of Central load.}$

$$W = W_f \quad \text{and} \quad W = Mg$$

$F_C = \text{centrifugal force acting on ball} = m\omega^2 r$

$T_1 =$ Force in arms

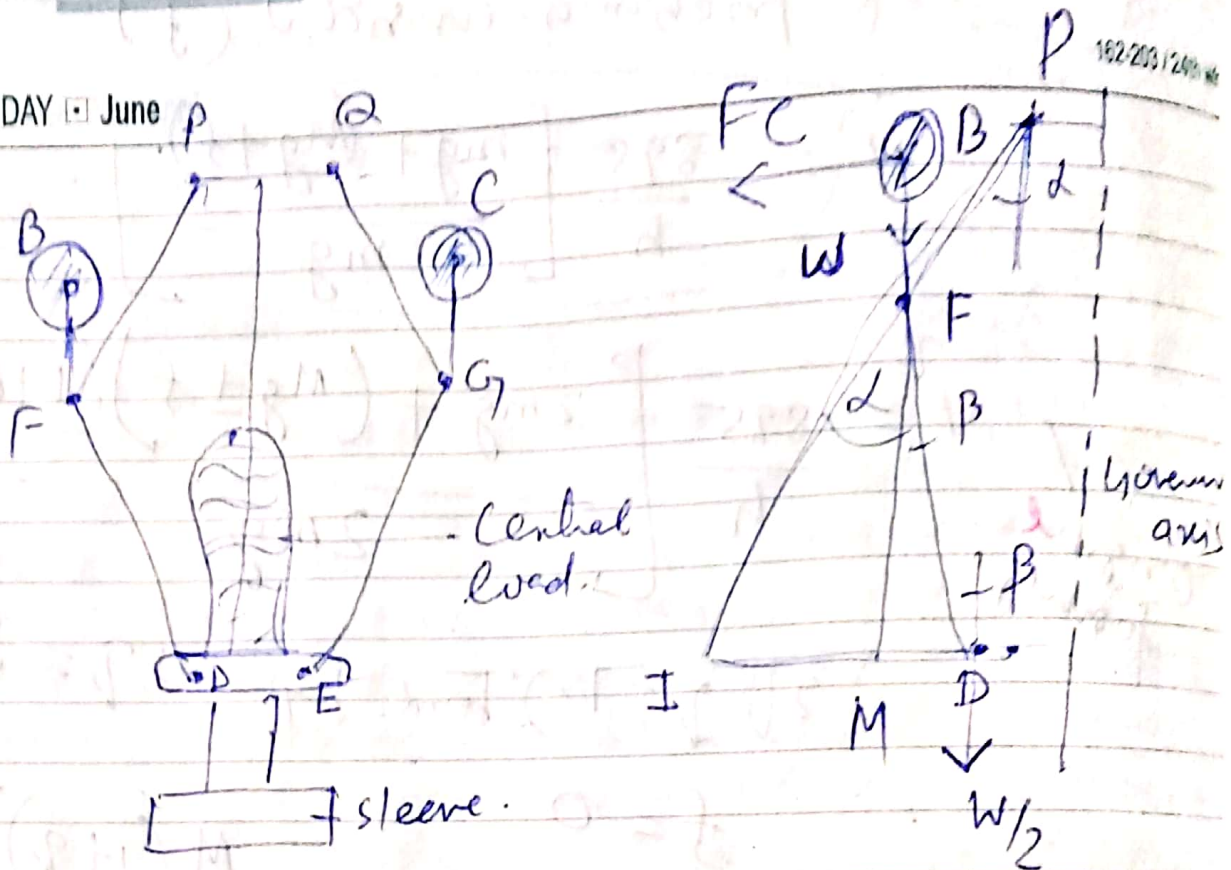
$T_2 =$ Fine in link

$\frac{1}{T_2} = \text{Force in link}$
 $\alpha = \text{angle of inclination of arm to vertical}$

angle of incidence θ $\rightarrow$ link to back

11

FRIDAY June



The Proell Governor has the ball fixed at B and C to the extending link DF & EG. The arms FP & GQ are pivoted at P & Q respectively.

Considering equilibrium of forces on one half of the governor. The instantaneous center (I) lies on the intersection of line PF produced and line from D drawn $\perp$ to spindle axis. The perpendicular BM is drawn on ID.

Taking moment about I, using the same notations

$$F_c \times BM = W \times IM + \frac{W}{2} \times ID$$

$$= mg \times IM + \frac{M \cdot g}{2} \times ID$$

Simplifying we get

$$F_c = mg \times \frac{I_M}{B_M} + M \cdot g \left(\frac{I_M + M D}{B_M} \right)$$

Dividing by F_M
 and putting $F_c = m \omega^2 r$, $\tan d = \frac{r}{h}$, $q = \frac{\tan \beta}{\tan d}$

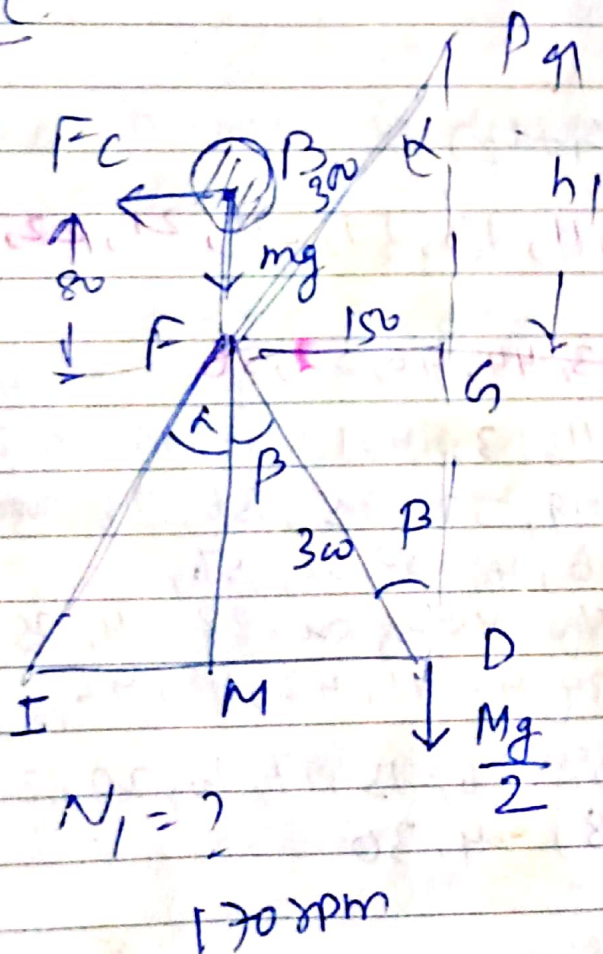
$$\checkmark N^2 = \frac{F_M}{B_M} \left[\frac{m + \frac{M(1+q)}{2}}{m} \right] \frac{895}{h}$$

14

MONDAY June

165-200 / 25th wk

Prob A proell governor has equal arms of length 300mm. The upper and lower end of arms are pivoted on the axis of governor. The extension arms of lower links are each 80mm long and parallel to the axis when the radii of rotation of the balls are 150mm and 200mm. The mass of each ball is 10kg and the mass of central load is 100kg. Determine the range of speed of governor.

Sol

next figure.

150 $\rightarrow$ 200height h_2 $N_2 = ?$

180 rpm

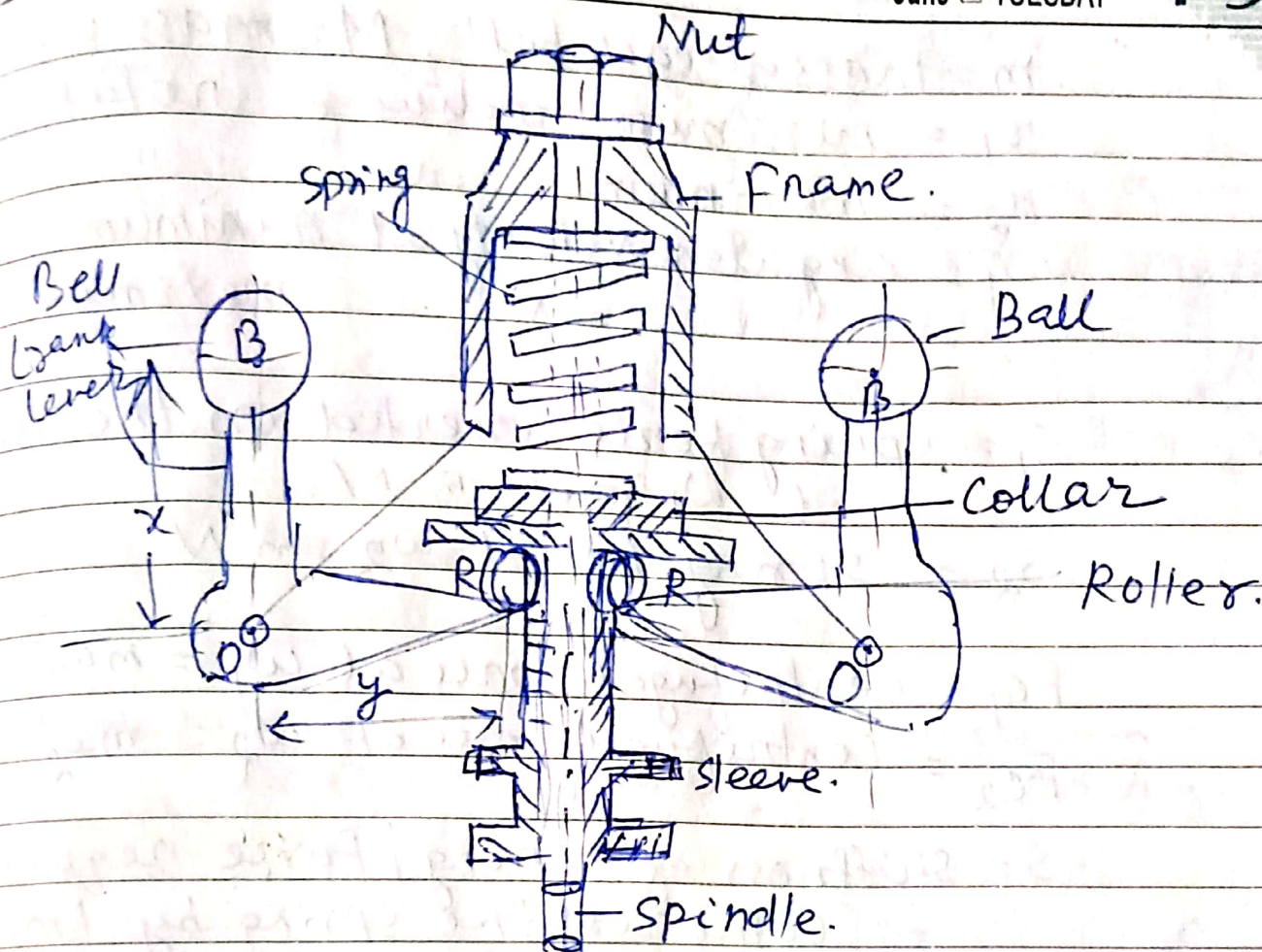
 $N_2 - N_1 = 10 \text{ rpm}$ $N_1 = ?$

170 rpm

HARTNELL GOVERNOR

June TUESDAY

15



It is a Spring loaded Governor. It consists of 2 bell crank lever pivoted at the points O, O to the frame. The frame is attached to the governor spindle and therefore rotates with it. Each lever carries a ball at the end of the vertical arm OB and roller at the end of horizontal arm OR. A helical spring in compression provides equal downward forces on the two rollers through a collar on sleeve. The spring force may be adjusted by screwing a nut up or down on the sleeve.

m = mass of each ball, M = mass of sleeve

r_1 = minimum radius of rotation

r_2 = maximum " "

ω_1 = angular velocity at minimum speed

ω_2 = " " maximum " "

S_1 = Spring force exerted on the sleeve at ω_1 in N.

S_2 = Spring force at ω_2 in N.

F_{c1} = Centrifugal force at $\omega_1 = m\omega_1^2 r_1$

F_{c2} = Centrifugal force at $\omega_2 = m\omega_2^2 r_2$

S = stiffness of spring / Force required to compress the spring by 1mm

x = length of vertical on ball arm of lever in m

y = length of horizontal on sleeve arm of lever in m

r = distance of Fulcrum O from Governor axis on radius of rotation when the Governor is in mean position in m.

$$r = r_1 = 0.12 \text{ m}$$

$$h = (r - r_1) \frac{y}{x}$$

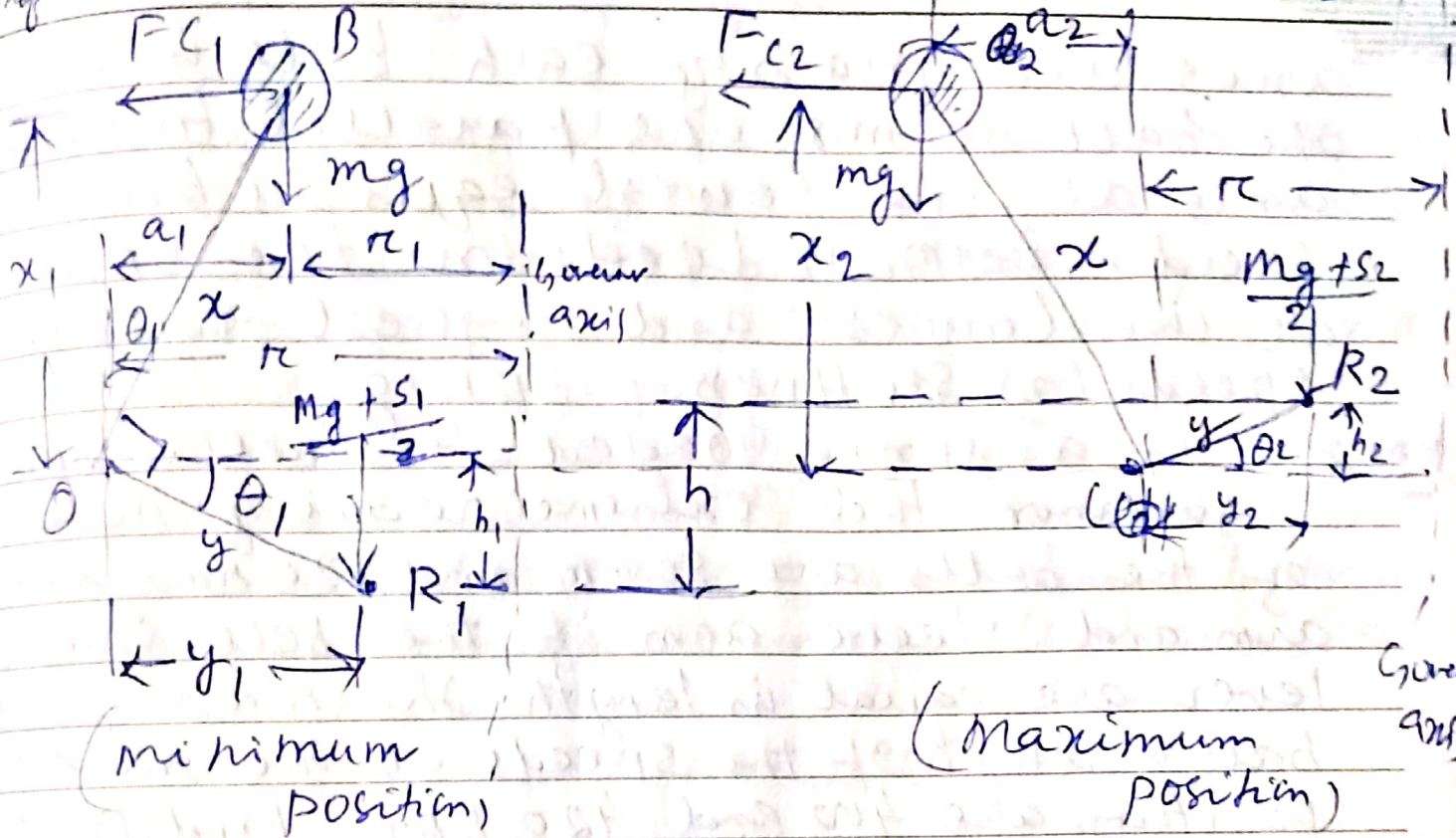
$$r_2 = ?$$

$$Mg + S_1 = 2 F_{c1} \times \frac{x}{y} \quad (1) \quad S_1 = \text{Spring force exerted}$$

$$Mg + S_2 = 2 F_{c2} \times \frac{x}{y} \quad (2) \quad S_2 = \text{Spring force exerted at } x_2$$

June THURSDAY

17



$$S = \frac{S_2 - S_1}{h} = 2 \left[\frac{F_{c2} - F_{c1}}{r_2 - r_1} \right] \left(\frac{x}{y} \right)^2 \quad \checkmark \quad x_1 = x_2 \times \frac{y_1}{y_2}$$

$$h = (r_2 - r_1) \frac{y}{x} \quad \checkmark$$

Prob. - A hartnell governor having a central sleeve spring and two right angled bell crank levers moves between 290 r.p.m and 310 r.p.m for a sleeve lift of 15 mm. The sleeve arm and the ball arms are 80 mm and 120 mm respectively. The levers are pivoted at 120 mm from the governor

axis and mass of each ball is 2.5 kg. The ball arms are parallel to governor axis at the lowest equilibrium speed. Detm. (1) Loads on the spring at the lowest and highest equilibrium speeds (2) Stiffness of spring.

Prob. 2 In a spring loaded Hartnell type governor the extreme radii of rotation of the balls are 80 mm and 120 mm. The ball arm and sleeve arm of the bell crank lever are equal in length. The mass of each ball is 2 kg. If the speeds at the two extreme positions are 400 and 420 rpm. Find (1) the initial compression of the central spring (2) Spring Constant.

Sol. $r_1 = 80 \text{ mm} = 0.08 \text{ m}$, $r_2 = 0.12 \text{ m}$, $x = y$, $m = 2 \text{ kg}$
 $N_1 = 400$, $\omega_1 = 41.9 \text{ rad/s}$, $N_2 = 420 \text{ rpm}$,
 $\omega_2 = 2\pi \times 420 / 60 = 44 \text{ rad/s}$

$$F_{c1} = m\omega_1^2 r_1 = 281 \text{ N}$$

$$F_{c2} = m\omega_2^2 r_2 = 465 \text{ N}$$

$$Mg + S_1 = 2 F_{c1} \times \frac{x}{y} \quad (M = 0, x = y)$$

$$\Rightarrow S_1 = 562 \text{ N}$$

$$Mg + S_2 = 2 F_{c2} \times \frac{x}{y} \Rightarrow S_2 = 930 \text{ N}$$

$$h = (r_2 - r_1) y / x$$

$$S = \frac{(S_2 - S_1)}{h} = 9.2 \text{ N/mm}$$

Initial Compression of

Central Spring

$$= \frac{S_1}{S} = \frac{562}{9.2} = 61 \text{ mm}$$

June SATURDAY

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SENSITIVITY — If there are two Governors

A & B running at same speed. When this speed increases or decreases by a certain amount, the lift of Sleeve of Gov. A is greater than the lift of Sleeve of Gov. B. It is then said that the Governor A is more sensitive than Governor B. So sensitiveness is defined as the ratio of difference between maximum and minimum equilibrium speeds to the mean equilibrium speed.

$$\text{So Sensitiveness} = \frac{N_2 - N_1}{N_1} = \frac{2(\omega_2 - \omega_1)}{\omega_1 + \omega_2}$$

STABILITY — A Governor is said to be stable when for every speed within the working range there is a definite configuration i.e. there is only one radius of rotation of governor balls at which the governor is in equilibrium. For a stable governor, if the equilibrium speed increases, the radius of governor balls must also increase.

ISOCRONISM — A Governor is said to be isochronous when the equilibrium speed is constant — (Range of speed is zero) for all radii of rotation of balls within the working range neglecting friction. The isochronism is the stage of infinite sensitivity.

For Isochronism $N_2 - N_1 = 0$ or $N_2 = N_1$

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So $h_1 = h_2$ which is impossible
for Porter Governor. Hence porter governor
cannot be isochronous.

Two thousand 10

172-193 / 2011 wt

FLYWHEEL A flywheel used in machine serves as a reservoir which stores energy during the period when the supply energy is more than the requirement and release it during the period when the requirement of energy is more than supply.

A flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed.

OR

Flywheel controls the speed variation caused by fluctuation of engine turning moment during each cycle of operation.

COEFFICIENT OF FLUCTUATION OF SPEED— The difference between maximum and minimum speeds during a cycle is called the maximum fluctuation of speed. The ratio of maximum fluctuation of speed to mean speed is called the coeff. of fluctuation of speed.

$$C_s = \frac{N_1 - N_2}{N} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} = \frac{V_1 - V_2}{V}$$

$m = \text{Coefficient of steadiness}$
 $= \frac{1}{C_s}$

Kinetic Energy of Flywheel

June TUESDAY

$$E = \frac{1}{2} mk^2 \omega^2$$

maximum Fluctuating Energy

$$\Delta E = \frac{\pi^2}{900} \times mk^2 \cdot N^2 \cdot C_s$$

Assignment

Prob. The mass of flywheel of an engine is 6.5 tonnes and radius of gyration is 1.8 m. It is found from turning moment

N/1142rpm diagram that the fluctuation of energy is 56 kN-m. If mean speed of engine is 120 rpm, find maximum and minimum speeds.

Prob. The flywheel of steam engine has a radius of gyration of 1m and mass 200 kg. The starting torque of steam engine is 150 N-m and may be assumed constant. Detm 1) angular acceleration of flywheel 2) the K.E. of flywheel after 10 s from start.

Prob. A horizontal cross compound steam engine develops 300 Kw at 90 rpm. The coeff. of fluctuating energy as found from turning moment diagram is to be 0.1 and fluctuating speed is to be kept within $\pm 0.5\%$ of mean speed. Find weight of flywheel required if radius of gyration = 2m. ($C_E = 0.1$, $k = 2$, $N = 90$, $P = 300 \times 10^3$)

$$m = 5630 \text{ kg}$$

$$\text{Fluctuation of speed } \pm 0.5\%$$

$$W = mg$$

$$\text{so } \omega_1 - \omega_2 = 1\% \text{ of } \omega$$

$$= 0.01 \omega$$

$$C_s = \frac{\omega_1 - \omega_2}{\omega} ; \Delta E = mk^2 \omega^2 \cdot C_s$$

DISTINCTION BETN. FLYWHEEL & GOVERNOR

Two thousand 10

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WEDNESDAY June

174-191 / 26th wk

FLYWHEEL

- 1) The function of flywheel is to decrease the variation of speed due to difference in input and output, as one or both of them vary due to cyclic fluctuation.
- 2) A flywheel is useful in regulating the speed during one cycle.
- 3) Mathematically flywheel controls $\frac{\Delta N}{\Delta t}$.
- 4) A flywheel stores up energy and gives up whenever required during a cycle.
- 5) A flywheel has no control over the quality of the working agent.
- 6) A flywheel is not an essential element of every prime mover. It is used only in case there is undesirable cycle fluctuation of energy output or input.

GOVERNOR

- 1) The function is to keep the speed of prime mover constant by adjusting the output of the engine to be equal to the external load in a given direction.
- 2) A governor regulates the speed from cycle to cycle i.e. over a no. of cycles of a prime mover.
- 3) Mathematically it controls ΔN .
- 4) It regulates the speed by regulating the quantity of working agent of prime mover.
- 5) It takes care of the change of quality of working agent.
- 6) A governor, being an adjuster of supply with demand, is an essential element of every prime mover.

JUN '10 01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

$$E = K \cdot \omega = \frac{1}{2} I \omega^2$$

$$\text{Torque} = I \cdot \alpha$$

175-190 / 26th wk

June THURSDAY

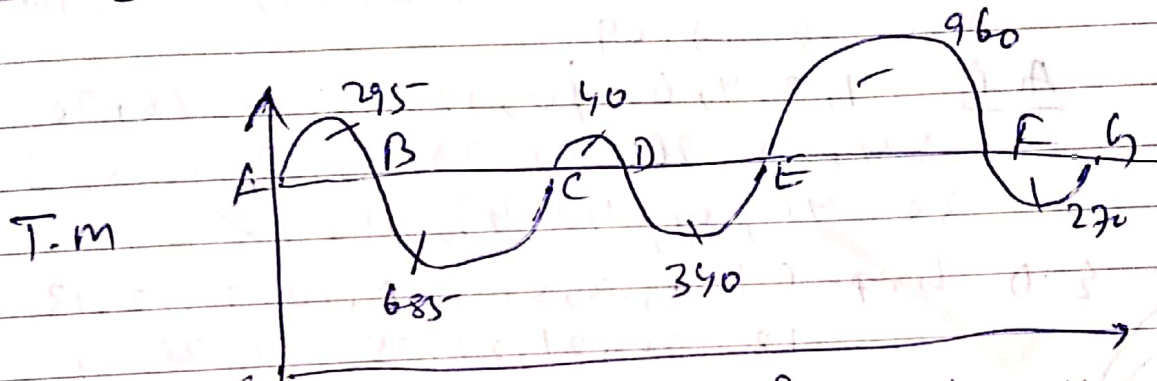
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Prob

The Turning moment diagram for a petrol engine is drawn to full scale,

Turning Moment $\rightarrow 1 \text{ mm} = 5 \text{ N-m}$, Crank angle $\rightarrow 1 \text{ mm} = 1^\circ$

The Turning moment diagram repeats itself at every half revolution of engine and the areas ^{above} ~~below~~ and ^{below} ~~above~~ the mean turning moment line taken in order are 295, 685, 40, 340, 960, 270 mm^2 . The rotating parts are equivalent to a mass of 36 Kg at a radius of gyration of 150 mm. Detm. C_s when engine runs at 1800 rpm.



[The area will be in mm^2 so $1 \text{ mm} \times 1 \text{ mm}$
Crank angle

Sol.

(B)

Let $E = \text{energy at A}$

$$A \rightarrow B = E + 295$$

$$C = E + 295 - 685$$

$$D = E + 295 - 685 + 40$$

$$E = E + 295 - 685 + 40 - 340$$

$$F = E + 960$$

$$G = \text{at F} - 270$$

$$\Delta E = \text{max} - \text{min} = - \text{mm}^2 + \frac{\pi}{36}$$

Quality of Governor

- 1) Sensitiveness - The definition of the term has not been standardised and hence varies from author to author. It is generally used to compare performance of 2 governors.

Defⁿ - change in level of balls for 1% change in speed.

$$2) \text{ sensitiveness} = \frac{\text{range of speed}}{\text{mean speed}}$$

- 2) Stability. A governor is said to be stable if it has one equilibrium speed, neglecting friction for a given configuration. In other words, an external force is applied on a sleeve of a governor running at a constant speed, it would change its configuration, but would immediately return to original one, as soon as the disturbing force is removed.

For spring controlled governors in which springs are compressed or extended within elastic limit, the controlling force curve would be a straight line.

$$F = ar + b$$

$$F = \text{controlling force}$$

$$a \neq b = \text{constant}$$

$$r = \text{radius of rotation}$$

$$\tan \phi = \frac{F}{r} = a + \frac{b}{r}$$

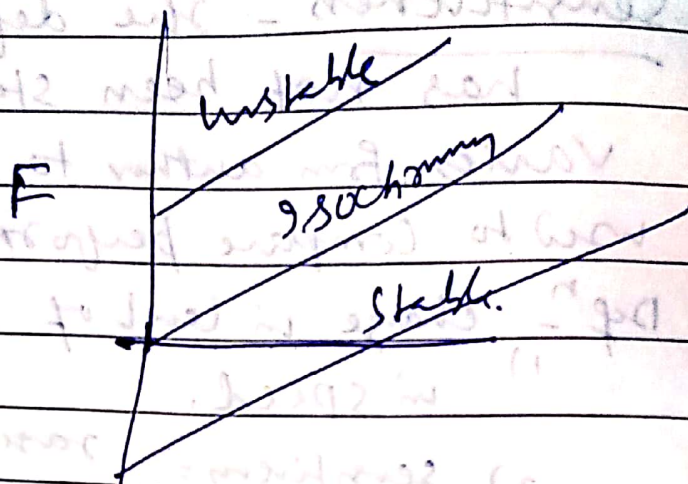
(2)

 $b > 0$ - unstable $b < 0$ - stable $b = 0$ - isochronous

classmate

Date

Page



An isochronous Governor runs at the same speed for different configuration.

Prob.

In a spring controlled Governor in which controlling force curve is a st. line, the balls are 0.35 m apart when the controlling force is 1200 N and 0.2 m apart ~~when controlling force is 600 N~~. At what speed the Governor will run when the balls are 0.25 m apart if each of them weighs 60 N? By how much the initial tension must be increased to make the Governor isochronous and what would then be speed of rotation?

sol

$$F = ar + b, \quad r = \frac{35}{2}, \quad r = \frac{20}{2}$$

$$1200 \text{ N} = 17.5a + b \quad (1)$$

$$600 \text{ N} = 10a + b \quad (2)$$

$$a = \frac{600}{2.5}$$

$$b = ?$$

$$F = m \cdot \omega^2 r$$

$$r = 0.25$$

(ii) if $b=0$

$$F = 8x$$

$$= 8 \times x$$

$$m\omega^2 x = 8x$$

$$\Rightarrow \omega = ?$$

$$N = ?$$

Effect of Governor

~~x~~
 ~~ω~~
 ~~$x=1$~~

~~Power~~
 ~~$= w(x^2 - 1) \cdot h$~~

When a governor is running at constant speed, the system is in equilibrium and hence the force acting on the sleeve is zero. However the load changes, the speed changes and hence sleeve changes its position. This can only take place if a certain force acts on it and it would occupy a newly steady position when the resultant force acting on it again becomes zero. (The Average force that acts on the sleeve for a given % change of speed is known as the effect of the governor)

$$\text{effect} = (x^2 - 1) \frac{w}{2}$$

effect of porter > simple watt.

< PTD

P-4

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Power. The power of a governor is defined as the W.D on sleeve for a given d/o.

If speed, Power = effort $\times$ displacement

$$\text{Power} = W \left(\frac{x^2 - 1}{x} \right)^2 \cdot h \quad \text{Watt}$$

$$= (W + w) \left(\frac{x^2 - 1}{x} \right)^2 \cdot h \quad \text{Other}$$

Balancing of different masses rotating in same plane has already been taught in the class and a problem relating to it had also been assigned.

Now Let us discuss the balancing of different masses rotating in different planes with illustrating with a problem. Look carefully how the problem has been solved by graphically. Because analytically it will be harder and cumbersome.

Problem

Four masses A, B, C and D are given in table below are to be completely balanced. The planes containing masses B and C are 30cm apart. The angles between B and C [Containing masses] is 90° . B and C makes angle of 210° and 120° respectively with D in the same sense. Find (i) the mass and angular position of A.

(ii) Position of planes A and D.

<u>Table mass</u>	<u>mass (kg)</u>	<u>Radius (cm)</u>
A	w	18
B	30	24
C	50	12
D	40	15

Solution

Now prepare a table as follows

Name of mass	mass in kg	Radius in cm	Force = $m \cdot g$	Distance from B	couple = $m \cdot g \cdot L$
A	w	18	$18w$	$-x$	$-18wx$
B	30	24	720	0	0
C	50	12	600	30	18,000
D	40	15	600	y	$600y$

Original Diagram according to Question

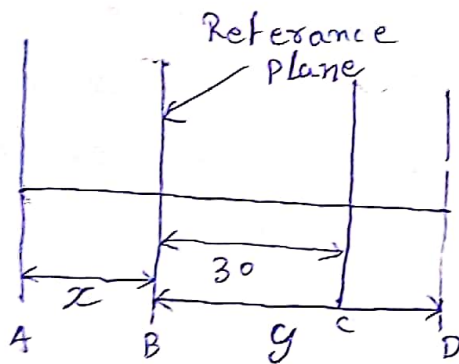


Fig-1

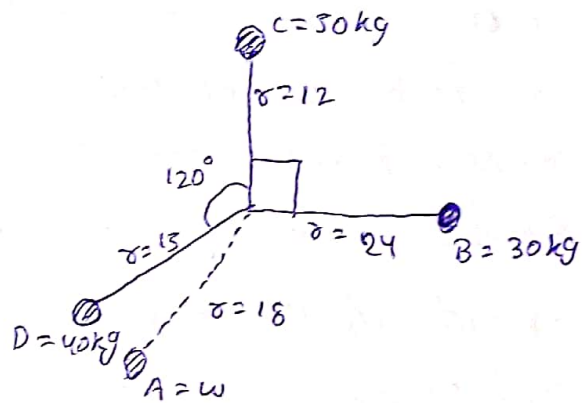
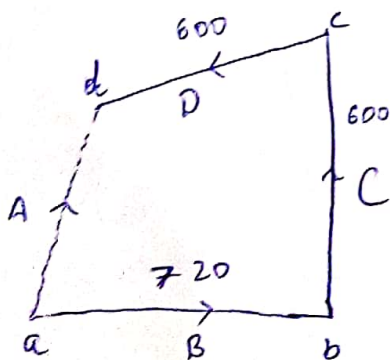


Fig-2

By taking a suitable scale draw forces diagram as below.



$$ab = 720$$

$$bc = 600$$

$$cd = 600$$

measure $ad = ?$

$$ad = 18w$$

$$w = ?$$

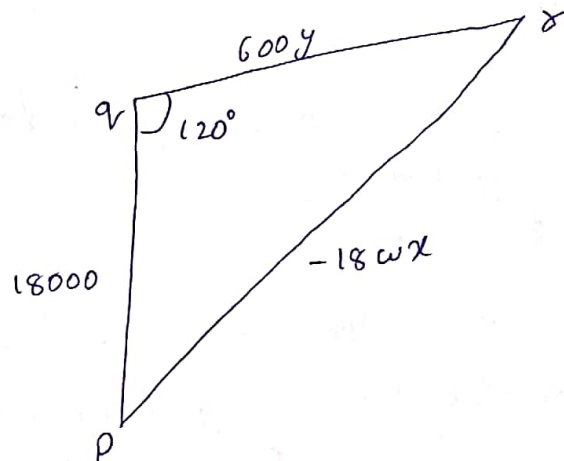
[force diagram]

To find angular position a line parallel to 1800 is drawn - in fig(2)

The mass A make _____ degree with mass D in anticlockwise direction.

Then draw the couple polygon to find α and γ

Draw a line parallel to mass C and equal to 18000 kg cm^2 to a suitable scale.



From two end of this draw two line parallel to masses A and D both meeting and closing the couple polygon By measuring

$$600\gamma = qx \text{ (measure)}$$

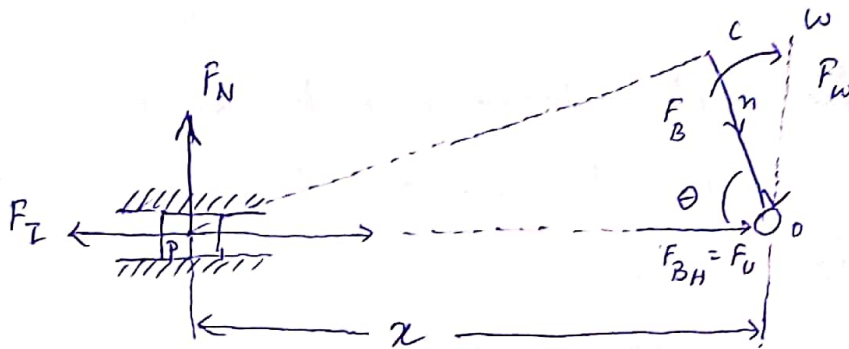
$$\gamma = ?$$

$$-1800\alpha = px \text{ (measure)}$$

$$\alpha = ?$$

Same ~~process~~ procedure will be adopted in balancing in practical (Tomlab)

Balancing of Reciprocating masses



Various forces acting on reciprocating parts of an engine

- ① Accelerating forces
- ② Inertial force

Resultant of all the forces acting on the body of the engine due to inertia forces only is known as unbalanced or shaking forces. If resultant of all the forces due to inertia effects is zero, then there will be no unbalanced force, but even then an unbalanced couple or shaking couple will be present.

Let F_R = force required to accelerate the reciprocating parts.

F_I = Inertia force due to reciprocating parts

F_N = Normal force

F_B = Force acting on crank shaft bearing

F_I is balanced by F_R

F_{BV} is balanced by F_N

Now $F_{BH} = F_U$ = unbalanced force

$F_N \times x = F_{BV} \times x$ shaking couple

Effect of reciprocating parts is to produce a shaking force and shaking couple, since the shaking force and shaking couple vary in magnitude and direction during the engine cycle, therefore they cause very objectionable vibrations.

Thus the main aim of balancing the reciprocating masses is to eliminate the shaking force. It is not usually practical to eliminate completely but reciprocating masses are partially balanced.

$$P_T = F_P = m\omega^2 r \cos \theta + m\omega^2 r \frac{\cos 2\theta}{n}$$

$$n = \frac{\text{length of connecting rod}}{\text{crank radius}}$$

$$F_P = \text{Primary unbalanced force} \\ = m\omega^2 r \cos \theta$$

$$F_S = \text{secondary unbalanced force} \\ = m\omega^2 r \cdot \frac{\cos 2\theta}{n}$$

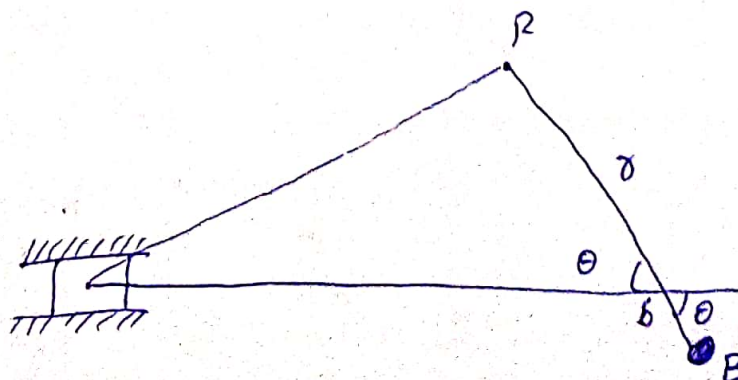
where m = mass of reciprocating parts

r = radius of crank

ω = angular speed

θ = crank angle.

Partial Balancing of Primary forces



(Balancing of Reciprocating parts by a rotating mass fixed opposite to crank)

Here $B \cdot b = R \cdot r$ — (1) for complete balancing

But reciprocating parts are not completely balanced, a fraction (C) is balanced. It may be $\frac{2}{3}$, $\frac{1}{2}$ etc. etc.

$$\text{so } C \cdot R \cdot r = B \cdot b \text{ — (1)}$$

so unbalanced forces along the line of stroke

$$= R \omega^2 r \cos \theta - B \omega^2 b \cos \theta$$

$$= (1 - C) \frac{R}{g} \omega^2 r \cos \theta \text{ — (2)}$$

Unbalanced force perpendicular to line of stroke

$$= B \omega^2 b \sin \theta$$

$$= \frac{C \cdot R \cdot r}{g} \omega^2 \sin \theta \text{ — (3)}$$

Resultant unbalanced force at any instant

$$= \frac{R \omega^2 r}{g} \sqrt{(1 - C)^2 \cos^2 \theta + C^2 \sin^2 \theta} \text{ — (4)}$$

0

Chapter - 6

VIBRATION IN M/C PARTS

Vibration

When elastic bodies such as spring, beam and shaft are displaced from mean position (equilibrium position) by the application of external forces and then released, a vibratory motion or vibration is produced.

Related terms used in vibration

Amplitude - Max^m displacement from mean position

Time period - Time taken to complete one vibration (cycle)

Frequency - No. of vibrations per second.

$\frac{1}{T}$ (Unit Hertz - Hz)

Cycle - One complete vibration or oscillation (motion completed during a period)

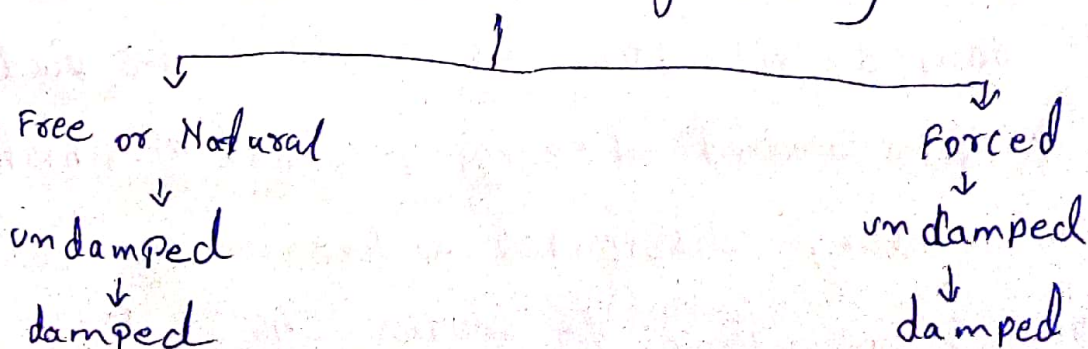
Types of vibration

They may be classified according to

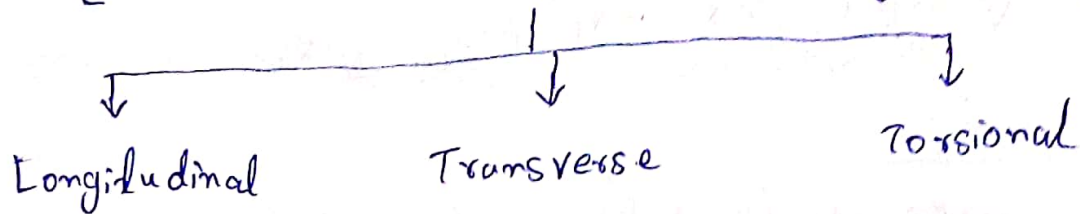
(a) the actuating force on the body

(b) the stresses in the supporting medium

[(a) Acc. to actuating force]



[(b) Acc. to the stress in the supporting medium]



Free vibration

Vibrations which are not actuated by any outside force are said to be free vibrations. As no external force, subsequent to the application of disturbing force acts, these vibrations should continue for ever as energy cannot be destroyed. This type of vibration is also known as natural vibration.

Forced Vibration

Vibrations which are caused or maintained by a periodic disturbing force are known as forced vibrations. As the energy supplied by the disturbing force gets supplemented ~~by the disturbing force~~ ~~gets~~ from time to time, this type of vibrations becomes a permanent source of trouble.

Damped vibration

When there is reduction in amplitude over every cycle of vibration, the motion is said to be damped vibration. This is due to the fact that certain amount of energy possessed by vibration system is always dissipated in overcoming frictional resistance to motion. As it is

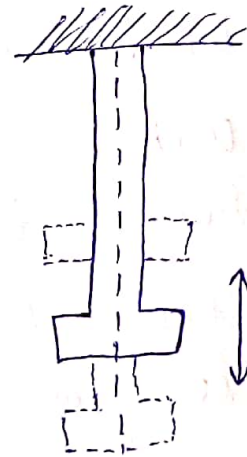
difficult to create perfect vacuum and also to have friction less motion. natural frequency does not occur in reality

Each of the above 3 types (Free forced and damped) can be further sub divided into 3 types.

- ① Longitudinal
- ② Transverse
- ③ Torsional

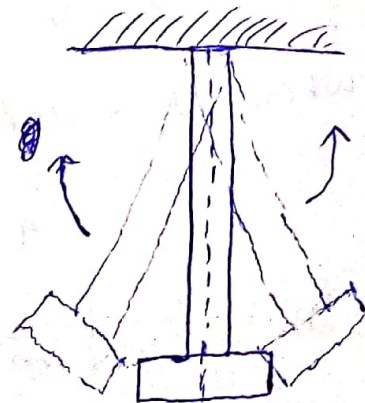
Longitudinal vibration

When particles of shaft move parallel to axis of shaft. In this case different particles constituting the body move in parallel plane. In this case tensile and compressive stresses are produced.



Transverse vibration

If the body instead of moving vertically up and down as in longitudinal vibration, in the exaggerated manner as

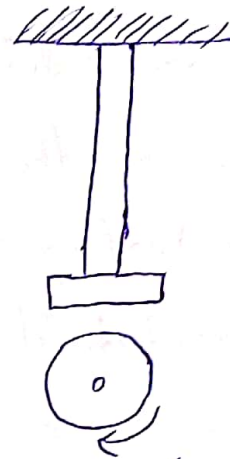


shown due to bending of supporting due to bending of supporting medium it is transverse. The ~~spring~~ spindle is subjected to tension and

Compression as a result of bending

Torsional vibration

If the particles of shaft move in a circle about the shaft (twisted and untwisted) the vibration is said to be torsional vibration. In this case torsional shear stress is produced.



Question

- ① What is vibration? classify it
- ② Write short notes on free, forced and damped vibration
- ③ Explain longitudinal, transversed and torsional vibration with diagram.

Formula for free longitudinal vibration

$$① F_n = \frac{1}{2\pi} \sqrt{\frac{g}{\delta}} \text{ Hz}$$

where δ = deflection

$$= \frac{W \cdot l}{A \cdot E}$$

$$② F_n = \frac{1}{2\pi} \sqrt{\frac{s}{m}}, \text{ where } s = \text{stiffness}$$

$$= \frac{\text{force}}{\text{deflection}}$$

m = mass of body suspended from constraints

Formula for Free Torsional vibration

$$f_n = \frac{1}{2\pi} \sqrt{\frac{q}{I}}$$

where q = torsional stiffness

$$= \frac{CJ}{L} \rightarrow \left(\frac{I}{\theta} = \frac{CJ}{L} \right)$$

C = shear modulus

J = polar m. I

L = length

Problems

① A shaft of 100 mm ϕ and 1m length has one end fixed and other carries a mass of 500 kg at a radius of gyration 450 mm, find natural frequency if $C = 80 \text{ GPa}$

② A cantilever shaft 50 mm ϕ and 400 mm long has a disc of mass 100 kg at its free end. $E = 200 \text{ GPa}$. Determine the longitudinal frequency

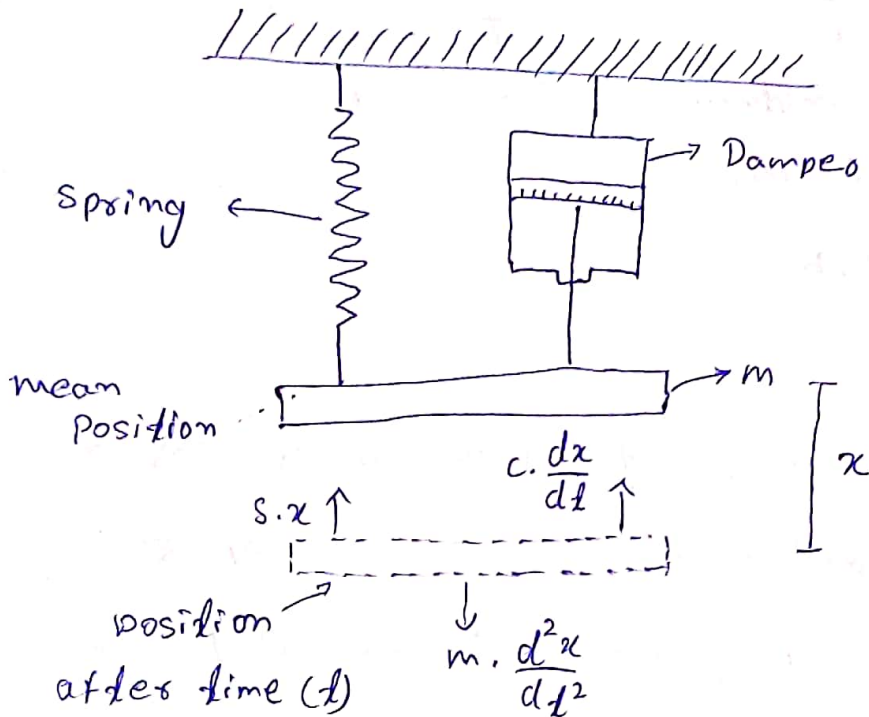
critical or whirling speed of a shaft

The speed at which the shaft runs so that the additional deflection of the shaft from the axis of rotation becomes infinite, is known as critical or whirling speed.

$$N_c = \frac{1}{2\pi} \sqrt{\frac{g}{\sigma}} = \frac{0.4985}{\sqrt{\sigma}} \text{ r.p.s}$$

same as Natural frequency but unit is r.p.m

Frequency of Free Damped vibration (viscous Damping)



In vibrating system the effect of friction is referred as damping. The damping provided by fluid resistance is called viscous damping.

m = mass suspended from spring

s = stiffness

x = displacement from mean position

s = static deflection of spring

$$= \frac{m \cdot g}{s}$$

c = Damping coefficient

$$= \frac{\text{Damping force}}{\text{velocity}} \quad \text{N/m/s}$$

$\frac{dx}{dt}$ = velocity , $\frac{d^2x}{dt^2}$ = acceleration

Damping factor / Damping Ratio

$$= \frac{C}{C_c} = \frac{\text{damping coefficient}}{\text{critical damping coefficient}}$$

$$C_c = 2m\omega_n = 2m\sqrt{\frac{s}{m}}$$

$$a = \left(\frac{C}{2m}\right) \quad [a \text{ constant}]$$

Over damping (large damping)

$$\left(\frac{C}{2m}\right)^2 > \frac{s}{m}$$

Under damping (small damping)

$$\frac{s}{m} > \left(\frac{C}{2m}\right)^2$$

Logarithmic decrement

$$\delta = \frac{2\pi C}{\sqrt{C_c^2 - C^2}}$$

ω_n = Circular frequency of undamped vibration

$$= \sqrt{\frac{s}{m}}$$

ω_d = circular frequency of damped vibration

$$= \sqrt{\omega_n^2 - a^2}$$

frequency of vibration of system

$$f_d = \frac{\omega_d}{2\pi} \text{ Hz}$$

Assignment

Q A vibrating system consists of mass of 200 kg , a spring of stiffness 80 N/mm and a damper with damping coefficient of 800 N/m/s . Determine

- (i) Frequency of vibration of system
- (ii) Critical damping coefficient
- (iii) Damping factor
- (iv) Logarithmic decrement

Causes and Remedies of vibration in a mechanical system.

Causes

Vibration can occur due to one or more factors at any given time. The most common is imbalance, misalignment, wear and looseness. ~~where~~ when the unbalanced weight rotates around the machine axis, centrifugal force is created. Due to occurrence of resonance also vibrations occur. (When a rotation frequency coincides with the resonance frequency of the machine, resonance occurs). It has a major impact. Bearing Damage also makes vibration. Damage of wear out gears also make vibration.

Remedies

Vibration cannot be completely remedied but can be controlled. The following precautions help to reduce whole body vibration exposure

- (a) Limit the time spent by workers on a vibrating surface
- (b) Isolation of vibrating source mechanically
- (c) Equipments are well maintained to reduce vibration.
- (d) Installation of vibration damping seats or springs to control vibration.